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MATHEMATICAL APPROACH TO PĀṆINI

(With special reference to first chapter of third book)

ABSTRACT

A THESIS SUBMITTED TO
THE ALIGARH MUSLIM UNIVERSITY, ALIGARH
FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN

SANSKRIT

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Summary.

(1) Introduction —

Many subjects are being st-udied at school and college level. Are they totally different from each-other? Are Chemistry and physics, two subjects? But, before no--dding, one should be aware that - Physical - Che--mistry and Chemical - Physics are also their par--ts. It is, due to the fact that 'Knowledge is a singular - unit.

Able persons observe the diff--erent possible aspects of a subject, they analyse them and then express in different forms. The fact and its expression are two utterly different as--pects. One fact may be expressed in two or more different ways. Mathematicians express their ideas partly in simple language of usage and partly in the language of symbols and signs.

The relation, between two subje--cts depends upon their peculiar nature. Here, Mathematics and Grammar — both are of Scienti--fic nature. Therefore, these two may be inter-- related.

If it is so, then, it will be easy to represent the rules of the great Sanskrit Gram--marian Pāṇini in mathematical - form

2 Mathematical Device —

In Mathematics, sym· and symbols are used at every step +, −, −, ×, ·, · etc are very common ones. Some other typical symbols are used in different bran·-ches of Mathematics These symbols are develo·-ped by Mathematicians to suit their needs of clarification of the concept. For, the study of Gram·-mar we should also develop few new ones. So·-me of them may be —

(1)

V	=	Vowel
C	=	Consonant
$\sqrt{}$	=	root
$\sqrt{}_V$	=	root, beginning with vowel
$\sqrt{}_{V=1}$	=	root, having only one vowel
a	=	affix
P	=	prātipadika
P_x	=	Prātipadika (P) + Case affix(es)
	=	Subanta
$\frown$	=	takes
$\overset{\cdot}{\frown}$	=	takes optionally.

There are some basic concepts in Mathematics. The concept of Set is one of them. A set is any c·-lass or collection of objects. The set and its objects are denoted by small and capital letters of English alphabet 'R' is a set and 'r' its obj·-

-ects 'x' is the object (or member) of set R. It will be denoted as — $x \in R$.

The set can be expressed in two forms — the tabular form and the set-building form. In the first, we enlist the objects and put them into-bracket { }. For example, if 'V' is the set of vo-wels of English alphabet, then —

$$V = \{ a, e, i, o, u \}$$

In the second form, we display a typical el-ement and equate it to the common property of the objects. The form of above set will be like this —

$$V = \{ x : x \text{ is the vowel of English alphabet.} \}$$

If the counting of objects comes to an end, then, it is called — 'Finite set', otherwise it is 'Infini-te one.' There is also a 'Null-set' which cont-ains no object. All or some objects of a giv-en set may constitute other sets. These are ca-lled — Subsets.

Let,

$$M = \{ a, b, c, d \}$$

$$N = \{ a, b, c \}$$

Then, N, is the subset of M. It is denoted by.

$N \subset M$.

If the objects are arranged in rectangular form constituting M rows and N columns, then it is called --- 'Matrix'.

It is written as - (if Matrix is A)

$$A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix}$$

Its general element is denoted as - a_{ij}, where 'i' and 'j' indicate row and column respectively.

If a Matrix is along with horizontal line, then it is called - Row Matrix, and if along with vertical Column, then - Column Matrix.

Sometimes, we discuss the numbers between two definite points. These are called - Intervals. It is open or close according to the ending points excluded or included.

Mathematically, they are —

$$(a, b) = \{ x : a < x < b \}$$

$$[a, b] = \{ x : a \leq x \leq b \}, \text{ respectively}$$

It may be open at one end and close at other. If Matrix is used, then some additional informations may be provided at the side or at the corner

3. Mathematical Representation —

If we want one or a few rules in the form of a unit on the base of a particular function to represent in Mathematical form, we must collect our raw material from Aṣṭādhyāyī, Mahābhāṣya, Kāśikā, Nyāsa, Siddhānta-Kaumudi etc., and then scrutinize the facts. After that we should try to express them in Mathematical form. For example, the facts related to the attachment of 'San' affix are—

(1). a— Affix San is added after the roots- Gup, tij and Kit. (3.1.5)

b— San is added only when they express nindā. Kṣamā and Vyādhi Prathkāra, respectively. (Kv. 343-4)

c— Above statement is general one, because we have exceptions of it. (Pa. 343, M. 344)

d— In the fifth rule Pāṇini has unaccented indicatory letters to show that they take ātmanepada affix (K. 344, mv. 20)

(2) a— 'a' of San is not indicatory. (M 345)

b— n of San indicates that accent of root loses its force. (Pa. 344)

(3) a— Roots man, badh, dān and San take affix San and the vowel of abhyāsa becomes dīrgha (3.1.6)

b— The affix is added, when the roots mean — to investigate, to doubt, to strengthen and to sharpen

respectively (K.)

4 (a) The affix san is added to any root which is expressing the object wished for and having the same agent as the wisher ture of. here the sense is - wishing and the process is optional (3.1.7)

(b) The affix san is added to any root which is expressing the sense of imminent danger. (K.S.' But Pāñjali debars it.

(c) The affix san gets the designation of ārdhadhātuka in the 7th aphorism only. (K. 351)

(d) A root can not take affix san to denote - wi -shing, if it has taken the affix. in the same sense, prezeiously. (Mv, Kv)

Here, we notice that there are three main elements — root, meaning and affix.

Let,

R = Set of Roots

r' = Root, which is the object of root 'is' and both the roots having same agent. It obeys the rules ordained in the adhi-kāra of dhātu.

R' = set of r' — roots.

Now, let — R = [Gup^a, tij^a, kit^a, mān^a, badh^a, dān, s'ān, r', R]

M = [to ~~denote~~ despise, to endure, to heal generally, to investigate, to loath, to st -aughten, to sharpen, to wish, the immin -ent danger.]

A San $\rightarrow Sa_1^5$ [san takes the form of sa' digit 1' indicates that the accent of root is not reduplicable and 's' indicates that padasamskaaming rules are indicated seperately]

y^1 = vowel of reduplicable syllable of the root.

$[Vi]_1^7$ = limits of i' are from 1 to 7.

Now, the representation is —

$$[\,[\,ri \frown a\ mi\,]_1^7\ j \frown am_j\,]_8^9\ [V_i^1 = T]_4^7 - A$$

$$\cdot\ [\,ri \frown am,\,]^a \quad — \ p$$

In the Matrix-form. —

$$\begin{bmatrix} Gup^a \\ Tij^a \\ Kit^a \end{bmatrix} \quad \begin{bmatrix} \text{to despise} \\ \text{to endure} \\ \text{to heal, generally} \end{bmatrix} \frown \left[san \rightarrow Sa_9^5\right] \rightarrow R$$

$$\begin{bmatrix} m\bar{a}n^a \\ budh^a \\ d\bar{a}n \\ s'an \end{bmatrix} \quad \begin{bmatrix} \text{to investigate} \\ \text{to bath} \\ \text{to straighten} \\ \text{to sharpen} \end{bmatrix} \frown \left[san \rightarrow sa_1^5\right] \rightarrow R$$

$|R|$ lit. same agent object $\frown$ $[San \rightarrow Sa_1^5] \rightarrow (R)$

$|(R)|$ $\uparrow$ $[San]$ Similarly, other facts can be represented

1 Mathematical Lexicie and Pāṇini —

Pāṇini uses sy
mbols in the form of indicatory letters, case affixes,
irreg. pronunciation and accent As in Mathematics, there
also, is much flexibility in them. 5th, 6th and 7th
cases are used in the sense of — 'immediately aft
-er', 'in place of' and 'immediately before'. Anunāsika
pronunciation of a vowel is for the sake of desi
-gnation 'it', which itself is of indicatory nature.
Accents 'svarita' are used to indicate – adhikār. Ātm-
-anepada affixes are employed after anudāttet 'it sa-
-mjñaka or indicatory letters are very common in
Pāṇini school of grammar.

We find – 261 Gaṇas in this
school. The collection of these is called - Gaṇapāṭha The-
-se are nothing but the set exhibited in Tabular-
form. Sarvādi Gaṇa is the first and Kṣubhnādi, the
last The set-builder form is also used by Pāṇini.
For example, the rule (1.1.1) = Vṛddhi Rādaic
or,

$$\text{Vṛddhi} = \bar{a}daic$$
$$= \bar{a}d + aic$$
$$= \bar{a} + ai + au$$

Thus,

$$\text{Vṛddhi} = \{ \bar{a},\ ai,\ au \}$$

There are many
more examples of such sets. There are sets in whi
-ch the number of element is fixed ie Counting
of elements comes to an end. Such sets are finite sets

But in ākṛti gaṇa, it is not so. Nāmadhātus are the parts of root-set, and the area of the use of some ~~text is very wide. So, one is unable to decide the nu~~ mber of roots Thus, the set of roots i.e. dhātupāṭha is an infinite set. Ākṛti gaṇa also fall in this category

Pūrvādi and tyadādi gaṇas are the parts of sarvādi gaṇa. In other words, we might say them - subsets.

Mathematically,

tyadādi ⊂ sarvādi

pūrvadi ⊂ sarvādi

The arrangement and description of root-affixes and case-affixes are of the nature of Matrix. The affix-es along with a particular vertical line denote the same number (i.e vacana). Here is the concept of Column - 1, 2 and n, where 'n' means either 3 or more than three. Similarly, the rows exhibit - 'Puruṣas' in root-affixes and 'Vibhakti' in case-affi-xes.

The words - 'maryādā' and 'abhividhi' are us-ed in the sense of - 'the limit exclusive' and 'the limit inclusive'. Here Pāṇini differenciates the two words clearly. The same concept is used as open and closed intervals in Mathematics.

5. Advantage of this Representation—

And in the last, here are two words about the advantage of this representation.

Firstly, any subject becomes more clear if it is studied from different angles. In our days, if any rule of Sanskrit grammar is explained in the same language it will be, then, clear to the less extent and to less persons, but the same rule will become more clear if translated in some other well-known language. Mathematical representation provides us another language for crystal clarity.

Secondly, we have taken up the 'unit' different from those rules. A particular function is regarded as the unit, here. Thus, we are able to understand the process to a greater extent.

Thirdly, the allied statements (e.g. reduplication of first syllable in the cases of San and Yan affixes) may be exhibited to widen the understanding of linguistic facts.

It might look a bit toil-some to the grown-ups, who find it difficult to mould themselves according to the new approach on one hand, and on the other, they understand the rules of Panini with much ease on account of their repeated use. But for a new-comer in this field, I hope, it will be very helpful, if explained by able persons.

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THESIS SECTION

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Certified that this work entitled as Mathematical approach to Panini (with Special Reference to 1st chapter of third Book) and submitted for the Ph.D. degree of Aligarh Muslim University is an original research work of Mr. Kedar Singh Bhartiya as regards language and diction.

4/6/83

(P.Anand)

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CHAPTERS

Preface

The ardent love for Mathematics had previously provoked me to do my M.A. in that subject and thereby to do some arduous work on Indian Mathematics. But, as the will of Supreme Power, whose finger I always think points the just, brought me here to have master's degree of Sanskrit. I, for the sake of genuine study of the subject chose grammar as special group.

After that, Gurudeva Dr. S. N. Misra being the torch-bearer took me to late Shri R. S. Tripathi. Knowing myself as student of Mathematics in B. A., he advised me to do work on present topic. Thus, in his ~~humble~~ noble guidance, I joined this university for doing the same work. But some adverse circumstances kept my speed very slow. Again, I lacked any literature in this yield to frame my mind except only one research article of Dr. M. D. Pandit from Poona.

Due to prolonged illness of Dr. Tripathi also, I could not keep required pace with my work. But the guidance of Dr. P. Anand filled me with zeal and enthusiasm, which enhanced my calibre. I was also rewarded the U.G.C. Fellowship and all the Gurudeva [illegible] their high handedness.

Thus, only within last eight months, I finished this work with full dedication, when all of a sudden I was asked to join at Lucknow as Asstt. Account Officer and was asked by my guide Dr. Anand to be a P.C.S. officer with the degree of Doctor.

Though the work was finished, still it was very difficult to get it typed as the key-board is incapable as far as new signs and symbols are concerned. Now copying the work in hand-writing (which might cause some difficulty in reading) and then getting the photo-state copies of it, I have to submit a bit, hurriedly. But, as for the contents, the emphasis throughout has been on clarification and evaluation. While the treatment is comprehensive and lucid, it in no way precludes a study of the original text. Long quotations have been eschewed as far as possible.

Again, as hard pressed of time I am, presence of few mistakes and difficulty in reading a manuscript, are not at all impossible. Hoping, that His Reverence will endure them, I place my work in your gracious hands.

June, 1983

K. S. Bhartiya

ABBREVIATIONS

A	=	Aṣṭādhyāyī
Aṣṭ or Aṣ	=	Aṣṭādhyāyī of Pāṇini — commented by S.C. Vasu
Av	=	Vārtika in Mahābhāṣya
K	=	Kāśikā
K-3 or K-III	=	The third volume of Kāśikā
M	=	Mahābhāṣya
Mv	=	Vartika of Mahābhāṣya
N	=	Nyāsa
Pa	=	Pada - Mañjarī
S.K.	=	Siddhānta - Kaumudi
S.K.-4	=	Fourth Volume of S.K.
V	=	Vārtika

1.

INTRODUCTION

Introduction

Almost all branches of knowledge are related to each-other. The fact is that the knowledge is a singular unit and the divisions are artificial. That is why we have not only the subjects like- Physics and chemistry but also Chemical-physics and Physical-chemistry. Similarly, Grammar, Literature and Philosophy etc. are inter-related.

There is surprising utility of Mathematics in classical, vedic and contemporary activities. It is also very useful in Prosody, Figures of speeches, Philosophy and Grammar etc.. As a matter of fact Mathematical-device plays an important role every where. Mathematical language is used not only in Physics, Chemistry and Astronomy etc. but also in the subjects of theoritical and descriptive nature like — Biology and Sociology. It enables us to attain breiwity, flexibility and clarity.

Mathematical Language: Just as there are various languages as - Sanskrit, Hindi, English, Greek etc., constituted by a number of combinations of sounds associated with meanings, so is there a language of Mathematics made of the digits- 0, 1, 2, 3 .... and symbols such as +, −, ×, ÷, = and many more yet to be described. Now, whatever be

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Ganita Sāra Samgrah — verses : 9-19. , chapter - 1
quoted in Hindu Ganita Śāstra Kā itihāsa P.P. 4.

these mathematical symbols, they are combined together to form numerals, equations and mathematical statements.[2]

In all the subjects its study is carried out to understand the relation among different things. Here lies the importance of the use of Mathematical language. Johnsons and others observe — "A practical reason for using the language of Mathematics is to show the connection between related quantities".[3] The following example would illustrate it —

A car is being driven at the average velocity of 80 Km./hour. Then it is obvious that it would travel 40, 80, 160 and 260 Kms. in half, one, two and three hours respectively. This fact can be expressed in simple language as 'the distance travelled by the car is the product of its velocity and the time taken in the transit', while in mathematical form as — $s = 80t$ where s = distance and t = time. It can be generalized. If v stands for velocity then, $s = vt$.

Fact and its expression.

One thing more is illustrated by above example i.e. the facts related to a particular subject are one thing and their expression quite another. ~~Any~~ Any fact can be expressed in more than one way.

It is very interesting to note that there was no math-

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2 Johnson and others. pp. 28

3 Johnson and others. pp. 32

-matical language in use in Europe before 1500 A D consequ-
ently mathematical statements had to be written in the lan-
-guage of the people. The rate of ~~mathematical~~ development of
mathematical ideas was also slow.

But then came the great upsurge in Mathematics and it was due to the development of special language for mathematics.[4] Thus it is obvious that the way of representing the facts plays very important role in the de-
-velopment of a subject.

The good representation needs the good un-
derstanding of related aspects of the subject and thus understanding itself depends upon the observation, experien-
-ce or the study of the subject. Thus a systamatic acquir-
-ement of facts and representation is very essential in a scientif-
-ic approach to a subject. 'A Science', according to Garner, may be described as a fairly unified mass of knowledge relating to a particular subject, acquired by systamatic observation, experience or study, the facts of which have been co-ordinat-
-ed, systematised and classified.'[5]

Grammar as a Science:

In the light of this description of science such as treated by Garner, we can say that Gram-
-mar, especially, is a scientific subject. (as is the view of Panini also) Grammar is based on systematic observatio-
-ns and classifications. It observes very minutely[6] the usage of speakers of concerned language and the litera-

4 Johnson and others pp 29.

5 Garner, Political Science and Government, 11

6 Mahabh ~~nAKṣmeṭaikĀ~~ vartate SutraKāreṇa (K-III 568)-4-2-14
Apo(u)pavidhi Off. Science exists such (ibid) The words 'akalāb

ture of the same very minutely. This observation is followed by analysis and classification. The word vyākaraṇ, equivalent of Grammar in Sanskrit language, itself means analysis. Apart from this, ample-evidences are available in Sanskrit Grammar-literature to prove above statement. Prakṛti, pratyaya, vikaraṇa, prātipadika and dhātu etc. indicate the analysis and classification of words. The well defined technical terms, the unique technique of description etc. are the indications of its scientific nature of description.

The forms of affixes, pratyāhāras, roots etc. are different in different schools of Sanskrit Grammar, which is due to the different angle of classification and expression of the fact.[1]

The unique method of Pāṇini.

We find a unique treatise in the form of Aṣṭādhyāyī which suggests that its author Pāṇini had classified the words, had noticed and analysised their inner functions and was able to sift-out some basic elements. These were grouped together in accordance of their similarity regarding a particular function. Roots were grouped in a special order to serve so many purposes. This set of roots is known as dhātupāṭha. Similarly, the sets of nominal stems, having a particular function — (i.e. a definite affix, long vowel, guṇa or vṛddhi process etc.) were formed.

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and 'gauptaḥ' are same in the form, but differ in accent on either side of the Ruvar-vipariṣā. Pāṇini notices it and gives two affixes aṅ and cin.

[1] For example — Maheśvara Sūtras 'ha-ya-va-rat' (6) and 'laṇ' (6) in Pāṇini-school but 'ha-ya-va-ra-laṇ' (5) in Chāndra School

He, for example, did not regard the word 'bhavati' as a unit in prakriyā-daśā. He analysis it as —

root bhū + affix śa + affix tip.

Here ū of 'bhū' is changed into 'o' (= guṇa) by guṇa-process and this 'o' into 'av' by the Sandhi-function

These rules are not stated at one place of Aṣṭādhyāyī. Tip is added by 3.2.123 and 3.4.78 and śap by 3.1.68. Guṇa is ordained by the rule 7.3.84, while avādeśa is by the force of aphorism 6.1.78. This gives a glimpse of the expression technique of Pāṇini.

All these sets and affixes contain some indicatory sounds. Ś, for example, of śap is for the sake of designation of sārvadhātuka, which itself is for the sake of guṇa-process.

In short, we can say that the facts are inter-related with each other. This relation must be noted very keenly and expressed systematically. This may be in the form of simple language or in the symbolic one. The best way of expression is to put the maximum in the minimum possible in terms of effort.

Here, it is obvious that Pāṇini himself used a language altogether different from the popular one, to express the linguistic facts and it must not be impossible to express them using new symbols and modern technique.

2.

MATHEMATICAL DEVICE

Mathematical Device

Symbols and notations in Mathematics

we are going to represent the aphorisms of Aṣṭādhyāyī mathematically, so the understanding of the basic symbols and notation is inevitable. It's a well-known fact that in mathematics we use symbols and notations to express our ideas in a compact form. Few important symbols are given below [1]

1. The Symbol '+' stands for 'addition' or 'plus'
   or + = addition or plus
2. The Symbol '−' stands for 'Substraction' or 'minus'
   or − = Substraction or minus

Similarly —,

3. $\times$ = multiplication
4. $\div$ = divisision
5. $\because$ = because
6. $\therefore$ = So
7. $:$ = ratio, such that
8. $\iff$ = implies and is implied
9. $\sim$ = Similar
10. $\subset$ = is subset of
11. iff = if and if only
12. S.t = Such that
13. $\exists$ = there exist
14. $\forall$ = for all
15. $\in$ = is member of

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1. (1 to 7) ... Common Signs
   (8-16) — A Text Book of Modern Algebra (96 99)

16. $\Longrightarrow$ = implies e.g. if $x = 4$, then $x^2 = 16$
mathematically $x = 4 \Rightarrow x^2 = 16$.
In other words $P \Rightarrow q$ means that the fact of P implies that of q

These symbols are developed by mathematicians to suit their needs. our needs while studying Grammar differ from these given. Therefore, we are to develop new ones. It would be afterwords in this very chapter.

In Mathematics, there are some basic concepts. 'The concept of set is fundamental in all branches of Mathematics.'[2] 'within the last century mathematicians have found in the basically simple notion of a 'Set' a unifying concept for entire field of Mathematics. We are going to take a look now at this modern approach which is to an ever increasing extent, influencing the language as well as the thinking of mathematicians, theoritical scientists and engineers.'[3]

'A set is any well defined class or collection of objects. By a well-defined collection we mean that there exists a rule with the help of which it is possible to tell whether a given object belongs or does not belong to the given collection'.[4] Here is an interesting example —
A barber in a town shaved all those who did not shave themselves and only those. S is the collection of such persons, now what is the position of barber? Is he a member of S? If he shaves himself as a man, he should-

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2 ibid — 98

3. Johnson and others — 181

4 A Text Book of Modern Algebra. 98

not shave as a barber if he does not shave himself as a man, he should shave as a barber" Here is a paradox, we are unable to decide whether barber is a member of S or not. So, S is a collection and not a set.

In set, the objects can be chosen from any source of supply. "There is no restriction on the nature of the objects."[1] These objects, constituting the set are called the element or the member of set. We shall denote the sets by capital letters and their elements by small letters. For example – if 'R' is the set of all roots then 'r' denotes any root. 'r' is the member of 'R' is denoted as – $r \in R$, and is read as 'r' belongs to 'R'.

A Set can be described in two forms – Tabular, and Set-builder form. In, tabular form the objects are enlisted which are seperated by comas and enclosed in brackets { }.

Just as for example – $A = \{1, 3, 5, 7, 9\}$

It denotes a set 'A' whose elements are 1, 3, 5, 7, 9.

In the set-builder form, we display a typical element and state the property which is satisfied by this element. For example, above set can be described as –

$A = \{x : x \text{ is odd numeral less then } 10\}$

To make it more clear, we give another example; Let V be the set of vowels of English alphabet X. The tabular form of it is –

$V = \{a, e, i, o, u\}$

[1] Johnson and others – 181

and 'Set builder' form is —

$V = \{ x : x$ is the vowel of English alphabet $\}$

Some Sets used in this work are —

$R = \{ x : x$ is a root $\}$

$P = \{ x : x$ is a prefix $\}$

$P = \{$ Pra, parā, apa, Sam, anu, avi, nis, nir, dus, dur, vi, ān, ni, adhi, api, ati, su, ut, abhi, prati, pari, upa $\}$

We use a vertical bar on the sign — '$\in$' to denote 'is not the member of': i.e.

$x \notin A$ means "x does not belong to A"

Null Set — There is a unique set called Null Set. It contains no member. Therefore, no example of it is available

Just as — $N = \{ x ; x \neq x \}$

Finite and infinite Sets —

A set is called finite or well-defined if its elements are listed especially or, its elements are described in a way that makes it possible for us to decide that how many elements belong to it. In other words, the counting of different elements comes to an end.[6]

But, if this counting process does not come to an end, then the set is called infinite.

R_1 and R_2 are two sets. Such that,

$R_1 = \{$ Gup, tij, Kit $\}$

$R_2 = \{ x : x$ is a root $\}$

6 A Text Book of Modern Algebra — H. Vasishtha

It is clear that the number of elements in R_1 set but we cannot say that how many elements are contained by R_2 set. So, they are finite and infinite sets respectively.

Subsets — Either all or few elements of a given set may constitute other sets. These sets are called subsets of basic set. In other words, if A and B are sets, such that every element of A is also an element of B, then A is said to be a subset of B.[1]

Let N_1, N_2, N_3 be three sets =

$N_1 = \{1, 3, 5, 7, 9\}$

$N_2 = \{9, 7, 5, 3, 1\}$

$N_3 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Here, every element of N_1 and N_2 is an element of N_3. Hence, N_1 and N_2 are subsets of N_3. It is written as —

$N_1 \subset N_3$ (i.e. N_1 is subset of N_3)

$N_2 \subset N_3$ (i.e. N_2 is subset of N_3)

It is clear that if elements of M and N are same, then, we can say that $M \subset N$ and $N \subset M$. Such subsets are called improper subsets.

Matrix — Order plays a vital role just as in our daily life, so in Mathematics also 08 10 P.M and 10 08 P.M are two different times. Similarly, in Co-ordinate Geometry (1, 2) and (2, 1) are the different points. If the

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[1] Ibid. - 107

members of a set are arranged in rows and columns we get matrix[8]. A set of ordered m n numbers (complex or real) arranged in a rectangular form, consisting m rows and n columns, written as —

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ - & - & - & \cdots & \cdots \\ - & - & - & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

— is called an $m \times n$ matrix. We can denote this matrix as $[a_{ij}]$ or by $[a_{ij}]_{m \times n}$. and the general element of it is denoted by 'a_{ij}'.

It must be noted that matrix contains ordered mn elements and not mn elements simply.

Thus —

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2}$$

and

$$\begin{bmatrix} b & a \\ d & c \end{bmatrix}_{2 \times 2}$$

— are two different matrices, simply because the order of elements is different.

Row Matrix — Any '$1 \times n$' matrix is called row matrix. In other words, if a matrix has only one row, then it is called a row matrix e.g.

$A = [1, 3, 5, 1, 9]$ and

$B = [9, 7, 5, 3, 1]$ are two different row matrix

---

8 Matrices — I

Here, $a_{1\times 2} = 3$

and $b_{1\times 2} = 7$

Closed and open intervals —— we often use the real numbers between two different points. These are called intervals.

Let $(a \text{ - - - - - - - - } b)$ be a set of real numbers, of which a is the lesser one. Then the set, $\{x : a < x < b\}$ is called open interval. Obviously, this set does not contain end-points. It is denoted by (a, b). closed intervals or the interval which contains the end-points also is denoted by $[a, b]$.

Thus, $[a, b] = \{x : a \leq x \leq b\}$

we can also have intervals closed at one end and open at other. e.g.

$[a, b) = \{x : a \leq x < b\}$.

and $(a, b] = \{x : a < x \leq b\}$

Use of Column-Matrix —— To make our representation easy to understand, we should use Row-Matrix. generally. we may call it a linear ordered set. The element in first column may be regarded as its first element. Thus, if R is the ordered set or row matrix then x_i means the i^{th} element of set R we shall write it as —

$$R = [x_1, x_2, x_3, x_4 \quad x_n]$$

in place of, $R = |x_{11} \quad x_{12} \quad x_{13} \quad x_{14} \quad x_{1n}|$

this expression is in Row Matrix

Similarly, in Column matrix —

$$R = \begin{vmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \end{vmatrix}$$

Sometimes one or many affixes are added after definite categories of roots to denote the same or different meanings. Here we find three elements — The roots (r), the meanings (m) and affixes (a). In ordered pairs they can be exhibited as — (r, m, a). If we have n roots having different affixes to denote distinct meanings, then we shall get n ordered pairs —
(r_1, m_1, a_1), (r_2, m_2, a_2) - - - - - - - - - (r_n, m_n, a_n).
If affix is same, ther pairs will be like this —
(r_1, m_1, a), (r_2, m_2, a) - - - - - - - (r_n, m_n, a),
and again if meaning in two cases is same then —
(r_1, m_1, a), (r_2, m_1, a) - - - - - - - - - (r_n, m_1, a).

To express it in Matrix form, we will have three matrixes. If r_1 r_2 - - - - - r_n m_1, m_2 - - - m_n and a_1, a_2 - - - - - a_n are the corresponding values, then the ordered pairs are — (r_1, m_1, a_1) (r_2, m_2, a_2) (r_3, m_3, a_3) - - - - - - - - (r_n, m_n, a_n)

And in Matrix form, $\begin{vmatrix} r_1 \\ r_2 \\ r_3 \\ \\ \\ r_n \end{vmatrix} \frown \begin{vmatrix} m_1 \\ m_2 \\ m_3 \\ \\ \\ m_n \end{vmatrix} \frown \begin{vmatrix} a_1 \\ a_2 \\ a_3 \\ \\ \\ a_n \end{vmatrix}$

while $\frown$ is the symbol for 'takes' and $\smile$ for to express it is possible that any row i.e. (x_1) may stand for single root or for a set of roots. For example, x_3 may mean a single root or a set of x_3', x_3'' and x_3'''. This would be expressed as —

$$\left| \begin{array}{l} x_1 \\ x_2 \\ x_3' + x_3'' + x_3''' \\ \vdots \\ x_n \end{array} \right|$$

similarly, if the meaning of x_3 set may be one or a set of corresponding meanings. The two conditions would be expressed as —

$$\left| \begin{array}{l} x_1 \\ x_2 \\ x_3' + x_3'' + x_3''' \\ \vdots \end{array} \right| \frown \left| \begin{array}{l} m_1 \\ m_2 \\ m_3 \end{array} \right|$$

and

$$\left| \begin{array}{l} x_1 \\ x_2 \\ x_3' + x_3'' + x_3''' \end{array} \right| \smile \left| \begin{array}{l} m_1 \\ m_2 \\ m_3' + m_3'' + m_3''' \end{array} \right|$$

We may not use suffixes 1, 2, and 3 etc. —

$$\left| \begin{array}{l} x \\ x \\ x' + x'' + x''' \end{array} \right| \frown \left| \begin{array}{l} m \\ m \\ m' + m'' + m''' \end{array} \right|$$

The number of row would indicate the related value of required element. However, when any one of the elements — root, meaning or affix — does not take different values i.e it remains the same for all other elements, it would be stated only once. for example ---

$$\left| \begin{matrix} x_1 \\ x_2 \\ x_3' + x_3'' + x_3''' \\ \vdots \\ x_n' \end{matrix} \right| \frown \left| \begin{matrix} m_1 \\ m_2 \\ m_3' + m_3'' + m_3''' \\ \vdots \\ m_n' \end{matrix} \right| \frown \quad [a]$$

Here affix 'a' is added after each root i.e. it does not differ with the difference of root.

Sometimes the indication of some other functions may be useful. It may be done at the right - (below) - corner. For example - 'yan' is added after a root to express repetition or intensity of the action. Then, its representation will be like this —

$$\left| x \right| \frown \left| \text{repetition} \mid \text{intensity} \right| \frown \quad [\text{yan}]$$

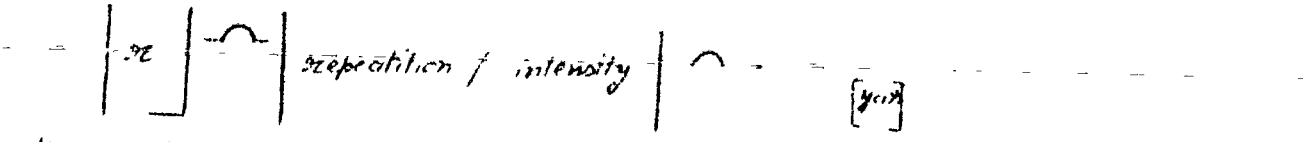
We know that bhyāsa - functions are carried out in the root. To denote it shall write —

$$\left| x \right| \frown \left| \text{repetition} \mid \text{intensity} \right| \frown \quad [\text{yan}]$$

↓

abhyāsa kārya

or without. the use of arrow ...



abhyāsa - Kārya

Useful Symbols and Notations.

Now we give a list of symbols and notations, which are necessary in our Mathematical - representation. These are —

1.	V	=	vowel
2.	C	=	Consonant
3.	√	=	root
4.	R	=	Set of roots
5.	√′	=	The form of root after addition of the augment
6.	√/m	=	Root '√' denoting the sense 'm'
7	^{C}√	=	Root, beginning with consonant
8.	^{V}√	=	Root, beginning with vowel
9.	√^{C}	=	Root, ending with Consonant
10.	√^{V}	=	Root, ending with vowel
11	√$_{V=1}$	=	Root, having only one vowel
12.	√^{ā}	=	Root, having ātmanepada affix
13	√^{P}	=	Root, having parasmaipada affix
14.	√^{u}	=	Root, having Ubhaya-pada
15	u	=	Affix
16	Sr	=	Sārvadhātuka affix
17	ar	=	Ārdhadhātuka affix
18	√ ∼ a/m		Root, having the affix 'a' to denote the sense of 'm'

20	a^s/a^r		Sārvadhātuka or Ārdhadhātuka [illegible]
21	a^k	=	Affix a expressing Kartṛ
22	$a^{k'}$	=	Affix 'a' expressing Karma
23	a^b	=	Affix 'a' expressing bhāva
24	$a^{kk'}$	=	Affix 'a' expressing Karmakartṛ
25	$a^k/k'/kk'$	=	Affix 'a' expressing K or K' or KK'
26.	lot^{sr}	=	Sārvadhātuka lot
27.	lot^{ar}	=	Ārdhadhātuka lot
28	lot^k (and so on)	=	lot expressing Kartṛ
29.	a/b	=	Action or bhāva
30.	$x^\frown/a^\frown m/\bar{a}r$	=	Root 'x' takes affix 'a' when Sārvadhātuka affix is to follow and takes optionally when Ārdhadhātuka affix is to follow
31	U	=	Upapada
32	A_1	=	Kṛta affixes or primary affixes
33	A_2	=	Subanta affixes or secondary affixes
34	W	=	Word
35	I	=	Indicatory part
36	S	=	Real part, similar to general affix
37	ds	=	Real part, dissimilar to general affix
38.	p	=	Prātipadika
39	Px	=	Prātipadika (P) + case affix (x) = Subanta
40	$\bar{P}_x$	=	Subanta used in comparison
41	P_1	=	Subanta [illegible] Kartṛ affix or Prathamā vibhakti
42	P_2	=	Subanta [illegible] Karma affix or Dvitīya vibhakti and so on
43	→gb	=	either guṇa [illegible] vṛddhi takes place
44	↛gb		Guṇa and Vṛddhi do not take place

45 ṁ/ṛ = Nasal of the root

46 ṁ/ṛ → ० = Nasal of the root is elided

47. ṁ/ṛ → ० = Nasal of the root is not elided.

48. ᶜ'ayo → ᵒ'iya = Last 'aya' changes into 'iya'.

49 ī ← v·ṛ = The first vowel will be replaced by ī

50 ऽऽ॰ ← ऽ॰ṛ = Reduplication of the first syllable.

51. ◠ = Takes

52 ◠· = Takes optionally

53 † = Does not take.

3.

MATHEMATICAL REPRESENTATION.

Mathematical - Representation.

In this chapter the aphorisms of Aṣṭādhyāyī would be represented Mathematically. We would take a particular process as unit rather than an aphorism. For example — 'Gup tij kidbhyaḥ san' is an aphorism. But there are two more aphorisms related to the process of attachment of San affix. So, we shall take all the three aphorisms as a unit.

1. Attachment of Affix San — The facts relating to this process are as follows —

(a) 1. Affix San is added after roots — gup, tij and Kit (A. 3.1.5)

2. San is added only when they express nindā, Kṣmā and vyādhipratikāra respectively. (K V 343-4)

3. Above statement is general one, for we have exceptions of it. (Pa 343; N 344)

(b) 1. In the fifth aphorism Pāṇini has unaccented indicatory letters to show that they take ātmanepada affixes. (K. 344, M V. 20)

2. 'a' of San is not indicatory (N. 345)

3. 'n' of San indicates that the accent of root loses its ^ (Pa 344)

(c) 1. Roots mān, budh, dān and s'ān take affix San and the vowel of reduplicatory syllable becomes dīrgha (A 3.1.6)

2. – Here again the affix is added when they mean to investigate, to look, to straighten and to sharpen respectively (K 346)

d. 1. — The affix San is added to any root which is expressing the object wished for and having the same agent as the wisher there of. Here, the person is wishing and the process is optional. (A. 3.1.7)

2. — The affix San gets the designation of 'ārdhadhātuka' in the 7th aphorism only. (K 361)

e. 1. — Affix San is also added to express the sense of imminent danger. (K.S.).

But Patañjali debars it.

f. 1. – A root can not take affix San to denote wishing, if it has taken the affix in the same sense previously. (Mv, Kv)

Mathematical Representation

Here, we notice that three elements are there, viz - roots, meanings, affixes.

Let,

R = Set of roots

r' = root which is the object of root 'iṣ' and both the roots have same agent. It obeys the rules ordained in the adhikāra of dhātu.

R' = Set of r' - roots.

Now let —

$R = [$ Cup^a, tiṣ^a, Kit^a, man^a, ~~kṛt~~^{a} budh^a, dān^a, śīn R', R]

Set of meanings, η

m = [to despise, to endure, to hurt generally, to investigate, to loath, to straighten, to sharpen, to wish, imminent danger.]

Λ = San $\longrightarrow$ Sa_1^5 [san takes the form of 'Sa.' Digit 1 indicates that the accent of root is not forcible and '5' indicates that pada governing rules are enlisted, seperately.]

Pāṇini uses San in its full-form at other places. So, the full form must be exhibited.

V' = Vowel of reduplicative syllable of the root.

$[Vi]_1^7$ = limits of i are from 1 to 7.

Now the representation is —

$$\left[\left[ri \frown a\, m_i\right]_{1}^{7} \frown a\, m_j\right]_{8}^{9} \quad \left[V_i' = \bar{I}\right]_{4}^{7} \text{ ---- A}$$

and

$$\left[ri \frown a\, m_i\right]_{8}^{9} \text{ ---------- B}$$

we read it as — 'ith' root takes affix 'Sa' to express 'ith' meaning and then it may take affix 'Sa' again alternately to express 'jth' meaning, and when — $[i = x : 4 \leq x \leq 7]$ than the value of reduplicate syllable is long i.

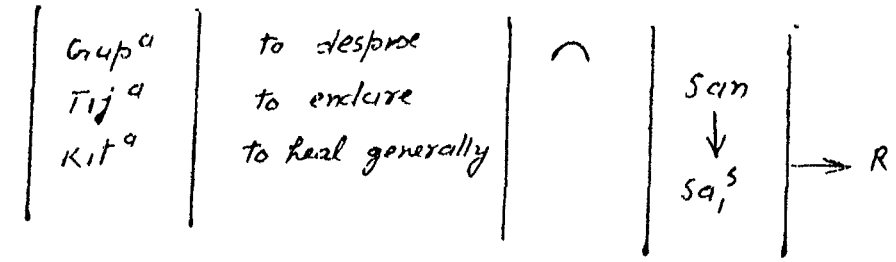
Let $i = 2$. Then root is $r_2 = tij$ and meaning is m_2 = to endure. Thus, it means that root tij takes affix Sa to express endurance. And this form — (tij + Sa) behaves like a root. In this form, if $J = 8$, then the meaning of 8^{th} is 'to wish'. It can take affix Sa again. In this case $V_i = \bar{I}$ is not applicable as i doesn't

fall within the closed interval [4,7]

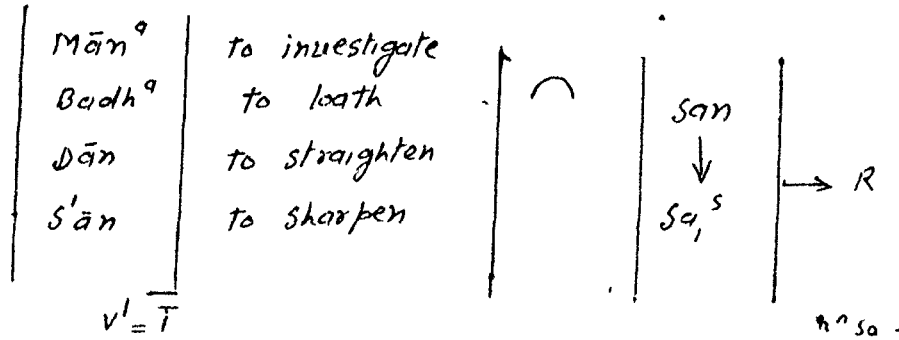
Representation B shows that when the root ~~is~~ is 8^{th} or 9^{th} element, then Sa may be added and but not more than it

Other Representation — The same results can be expre -ssed in graphical form also. as -

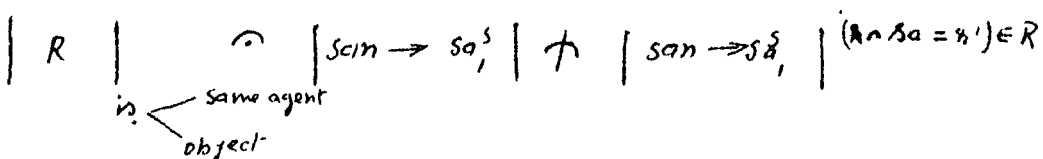
(A)



(B)



(C)



2 Nāma - dhātavaḥ —

Now we come to an important catagory of Affixes, which are added aft -er Subanta - word or prātipadika These are — kyac Kyaṣ, Kyan, Kāmyac, Kvip, ṇi, and, ṇic They're -m roots and these new ones are called. nāma dhātu

[illegible]

(b) affix 'kyac' is added optionally after such word which has taken an affix denoting object, to express that the agent treats the object for himself. (A 3.1 8, K 359)

(b) In the process of addition of kyac, pratipadikas must be stated as prātipadikas ending in 'm' and indeclinables. (v).

On Vedas, the affix is added even when the wish is with regard to another. (KV)

(b) The affix 'kāmyac' is also added in the same conditions. (A. 3.1 9)

The initial letter 'k' of kāmyac is not indicatory as it serves no purpose. (K 359)

(c) The affix 'kyac' is optionally employed after a word ending in a case affix denoting object, to express that the agent treats a particular person or thing like this object (A 3.1 10; K)

So also in locative case. (v)

(d) The affix 'kyan' is optionally employed after a word ending in a case affix, expressing karte, to denote the sense that 'the agent he behaves like'. The elision of final 's' of prātipadika takes place simultaneously, if it ends with a 's'

The elision of final 's' is a matter

-ble in the case of 'ojas' and 'apsaras' and optional in case of 'payas'. (KV, AV)

The word 'payas' stands for all the other possible cases. (Pa. 362)

In the same sense comes the affix 'Kvip' after the words like —avagalbha, klizna and hoḍā

According to some other Grammarians Kvip is added after all prātipadikas or crude nou-ns (KV).

(e) The affix 'Kyaṅ' is added after the crude form bhṛśa & c., if they do not end in 'cvi' affix, to denote the sense of becomin-g what this was not previously. There is elision of final consonant, simultaneously, if the word ends with that. (A 3.1.12)

(f) The affix 'Kyaṣ' is employed after Lohita & c and the words ending in dāc affix. [in the same conditions as mentioned above]

(A 3.1.13)

Lohita & c is a ākṛti gaṇa (K. 369)

(g) The affix 'Kyaṅ' is employed after the word Kaṣṭa ending in fourth case affix in the se-nse of exerting in dishonesty. (A 3.1.14)

This affix is added after the words — Śabda, kaṣṭa, Kakṣa, Kṛcchra and gahana in the sen-se of exerting in evil (V)

(h) The affix 'Kyaṅ' is employed after the words- 'Romanṭha' and 'Tapas', when they are used

as the objects of āvṛtti (repeating) and cara (performing) respectively. (3.1.15)

The affix is added when the sense is the movement of jaws (in case of romantha) (KV)

Tapas takes the parasmaipada affix (V)

(i) In the sense of ejecting the affix Kyan is added after the words 'vāṣpa' and 'ūṣmā'. (3.1.16)

So also after 'Fena' (KV)

(j) The affix 'Kyan' comes after the words — Śabda, vair, Kalaha, abhra, Kaṇva and Megha in the sense of making (A 3.1.17)

Sudina, durdina and nīhāra take the same affix to denote the same sense. (K v. AV-2)

[note — AV-2 means that two vartikas in A to this effect.]

So, is the case of aṭā, aṭṭa, Kaṭā, paṭā, Śaṭā and Kaṣṭa.

(k) The affix 'Kyan' is added after Sukha &c. to denote the sense — 'the agent feels that.'

(l) In the sense of doing (K, making), the affix Kyac is added after the words — Namas, varivas and citrañi (3.1.19)

'ñi' of citra is for the sake of ātmanepada affix (K. 37.6)

(m) In the sense of doing (K, making) affix ṇiṅ is added after the words puccha, bhāṇḍa and cīvara, when they are object of action (3.1.20) The senses are .. lifting or

throwing about, accum
-ring wearing.
sense of making, the affix
after munda, miśra, ślakṣṇa, lavaṇa, vṛt
vastra, hali, kali, kṛta and tūsta.

Pāṇini uses 'hali' and 'kali' instead of 'hala' and 'kala' to show that 'Sanhad-bhāva' does not take place while performing Aurist.

K. explains that Vṛta takes the affix to denote the sense either of eating or obtaining therefrom. Similarly, the senses are - Covering and taking, when the words are - (vastra) and (hali and kali) respectively.

ṣic is added to any prātipadika, having affix of Karma[1] (KV. 397) 2.

Now, it is obvious that these affixes are added after Subanta word. Affixes 'Sup' are added after prātipadikas[2]. Thus —

Subant Pada = Prātipadika + Sup = P + Sub
= P_x (Let it be so when x is any number)

These 'Sup' affixes denote Karma, Kartā, Sampradāna and adhikaraṇa.

We shall denote these Subanta - pada by - P_x where x is any number. Here 'P' stands for prātipadika and x for Sub. In the Kartṛ

---

1. (KV. 397)
2. (4.1.1.) and (4.1.2)

-Vācya (active voice) Kartṛ, Karma, Sampradāna and Ad-hikaraṇa are denoted by 1st, 2nd, 3rd and 7th Vibhaktiṃ we shall also use these numbers. Thus, P_1 means - prāti-padika having a sup- affix denoting Kartā. P_2 means - prā-tipadika having a sup affix denoting Karma etc.,

Sometimes these affixes are used after the simple subanta and sometimes are added after a word in comparison we will denote this upmāna prātipadika by - $\bar{P}$.

Representation — There are seven affixes in this cat-egory. we would represent the pro-cess in seven diagrams. But, we must show the gene-ral characteristics of all these affixes. Here, thus dia-gram will show them —

$P_x \frown$

Kyac	→	ya^b	°aya → īya (kyaci ca)
Kāmyac	→	$kamya^b$	
Kyaṅ	→	ya^a	°aya → āya (Akṛta sārva-dhātukyoḥ)
Kvip	→	0	
Kyaṣ	→	ya	°aya → āya (" ")
Ṇiṅ	→	$\dot{T}^a$, ^{a}	
Ṇic	→	i^b	

This representation clearly shows that to form a nām-dhātu, any subanta word can take any one of the seven Kyac & C affixes. The affixes are — kyac, kāmyac, kyaṅ, kvip, kyaṣ, ṇiṅ, ṇic. But these also con-tain the indicatory parts. Their remaining parts are ya, kāmya, ya, zero, ya, $\dot{T}$ i and i respectively

Here we find that ya is remaining part of these affixes and ī of two. But there is some difference among all of them, which is on account of particular affixes. The first ya takes parasmaipada only and second one takes ātmanepada only, while the third ya takes both of them. Similarly, first ī takes the ātmanepada and second ī parasmaipada only.

The last vowel 'a' changes when ya affixes are added.

When Kyac is added, $^{\circ}a \longrightarrow \bar{\imath}$

When Kyan or Kyaṣ is added, $^{\circ}a \longrightarrow {}^{\circ}\bar{a}$

Addition of Kyac affix —

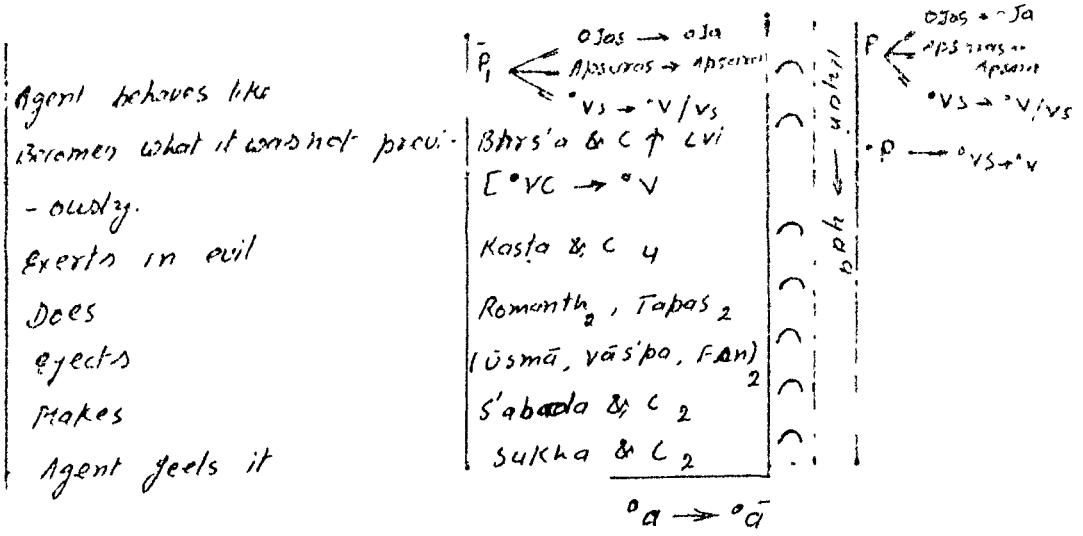
Agent wishes for himself Agent treats like Agent does	$P_2 - (^{\circ}m + ind)$ $\bar{P}_2 + P_1$ namas, varivas, citra^{c}	⌢ ⌢ ⌢	Kyac → ya

$^{\circ}a \longrightarrow {}^{\circ}\bar{\imath}{}^{\circ}\bar{\imath}$

Read it as — To denote the sense that the agent wishes for himself any Subanta word having Sup-affix denoting Karma, takes Kyac affix optionally. Here $\bar{P}_2$ denotes a Subanta word having Sup affix to denote Karma and used in comparison. chitra^{a} — shows that the ātmanepada affix must come after (chitra + ya). In the remaining cases parasmaipada affixes are added after [P + ya]. The final 'a' of pratipadika changes into long ī.

Here, P_2 – (°m + ind.) means that all prātipadikas but not those, which end in m and are indeclinable.

Similarly, other representations are –



Here – S'abda & C_2 = Sabda & C + Sudina & C + ata & C

Here, we have $\bar{P}_1$ ⟵ Ojas → Oja
Apsaras → Aprara
°Vs → °V/°Vs

It means that when $\bar{P}_1$ represents Ojas or Apsaras, then the final 's' is elected when the affix Kyan is added. But in the other cases, if Prātipadik ends with s' i.e its construction is – °Vs then it may drop final s or may not.

Similarly, the final consonant of bhrs'a & C is dropped, and it is shown by °VC → °V

Below, we have the Prakṛtivākya mahāvā $^{0}a \rightarrow ^{0}\bar{a}$
It simply means that the final a of Prātipadika takes the form of ā when kyan is added.

The representations of attachment of Ṇin and nic -

lifting or throwing about	Pṛccha $_2$	⌒	
Accumulation in heap	Bhāṇḍa $_2$	⌒	ṇin → $\bar{I}^{a}$
wearing or acquiring	cīvara $_2$	⌒	

and

Making Eating or obtaining therefrom Takes	mund & C_2 + P_2 Vṛta $_2$ Kali $_2$ Kali $_2$	⌒ ⌒ ⌒	d^{1} ← ṇiN	No Sambad-bhāva in Aurst

None of the remaining three affixes needs a seperate dealing. So we will represent them by one digram —

Agent wishes for himself	$\frac{P_2}{galbha}$	⌒	Kāmyac → Kāmya
Agent behaves like	^{a} & C + $\bar{P}^{p}$	⌒	kvip → 0
-Becomes what it was not Previously	lohit & (P + dac) ↑ cvi	⌒	Kyaṣ → yą

Here lohit & C means that it is a ākṛti gaṇa, and a bar over galbha and p shows that they are to use in comparison.

§ 4232 All these aphorisms can be given a formula-form Let the ordered set of Subanta be —

$$P = [P_1, \bar{P}_2, (\text{namas, vaivas citra}^{a})_3]$$

Ex. Set of senses be —

m_2 = [the agent (wishes) for himself , treats like (kartṛ)]

and set of affix —

[kyaC → ya]

Now the representation will be — —

$$(\because i=1)$$

$$m_i \; pi \frown (kyaC \rightarrow ya)\,[{}^{\circ}aya \longrightarrow {}^{\circ}\bar{a}ya]$$

pi is read as — to denote the ith meaning, ith prat.padika takes affix ya. In this process 'a' of pratipadika changes into $\bar{a}$, when $i=1$, the addition of affix is optional.

If we put $i=2$ then,

m_2 = Agents treats like,

$\bar{P}_2 = \bar{P}_2$ = A prātipadika, which has taken an Sup to denote object and is used in comparison

A = Kyac

$i \neq 2$ So, the process is not optional but nitya Thus, it means that A prātipadika in comparison with Sup affix denoting the object always taken affix Kyac to denote that the agents treat it like, and if the final letter of pratipadika is 'a', it is changed into $\bar{a}$.

Similar formulae can be formed for other affixes.

3. Yaṅ - affix —— The affix yaṅ is added to a root to express the repetition or the intensity of an action, generally. We are provided with following data by Pāṇini and his Commentators —

(a) The affix yaṅ, in the sense of repeation or intensity of the act comes after a root, having a single vowel and beginning with a consonant.[1] To denote this sense Pāṇini has used only one word Kriyā - Samabhihāra and the idea of Kāśikā Kāśyapa is that the word Kriyā - Samabhihāra means repeatition or intensity.[2]

The affix is added only to a simple root but not after a compounded root with up-sarga.[3]

Word Kriyā- Samabhihāra qualifies the root. So repetition or intensity is not the sense of affix, but of the root. This affix is added only to show this particular sense.[4]

The roots śubha and ruc takes this affix to show the repeatition of action only. (K, 358)

Intensive affix yaṅ comes after Śūci, Sūtri, Mūtri, oṭi, ṝ, asu and Ūrṇu. Though some of them contain more than one vowel and others begin with vowel.[5]

---

1 3.1.22

2 Paunaḥ Punyaṃ bhṛśārtho vā Kriyā samabhihāraḥ. (K. 379)

3 Saparsargan mā bhūt. (K. 380)

4. (K, 379) (N 379)
Kriyā hi dhātu vacyā. Samabhihāra - viśiṣṭapi kriyaiva bhavati (Pa 379)

5 (Kv. 382)

(b) The affix invariably comes in the sense of crookedness after a simple root expressing motion.[6]

Pāṇini has used the word 'Nitya' for the sake of 'Viṣaya - niyami' which means that the root expressing motion can have this affix only to show the crookedness of action but not in the sense of repetition or intensity[7].

However, Haradatta does not agree with this explanation of word - Nitya and he debars it as the same meaning can be expressed without this word. (Pa 384)

(c) The affix yan, when intended to convey the sense of contempt in respect to the sense of the root always comes after following roots. viz —

lup, sad, car, jap, jabh, dah, dams' and gṝ[9].

§ 4·302 Let,

$^{c}Rv=1$ be a root. Such that it begins with consonant and have a single vowel.

$\frac{R}{m}$ means that root R is denoting the sense 'm'. R^{m} stands for a root which denotes the motion.

lup & C = { lup, sad, car, jap, jabh, dah, dams', gṝ }

śūci & C = { śūci, śūlri, mātri, ali, ṛ, aṣ'u, ūrṇu }

---

6 (3.1.23)

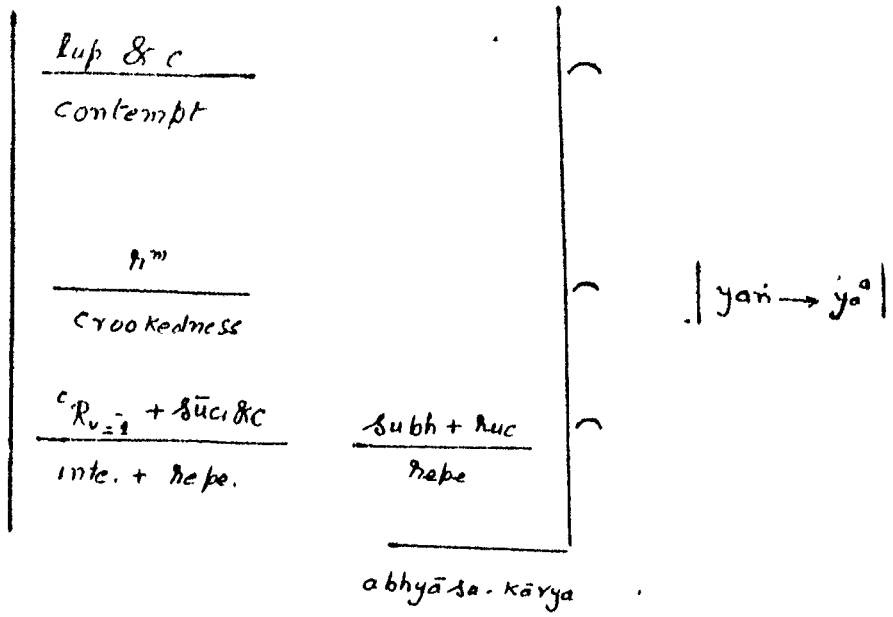
7. (K. 384)

8 Tādetannitya grahaṇaṁ cānitya prayojanam,

9 (3.1.24)

4 303 Representation

Now it is ——



Here, we do not denote any sense seperately after the affix yaṅ. As it is added in 'svārtha' i.e. does not alter the sense of root, but its addition shows that the root is denoting a particular sense.

4 Causative - Affix —

In the present section of Grammar, the explanation of causative form of verbs is given by the addition of affix ṇic. One and only one aphorism ordaining this phenomenon is — hetumati ṇic (3 1.26) Kāśikā Kāśyāpan comments — 'one, who inspires the subject is called hetu; his activities - sending etc. are — hetumān; to denote this hetumān the affix ṇic is employed.[1]

The meaning of 'hetu' is not so limited. Anything or any person can be hetu, if it is capable of inspiring the subject.[2] Nyāsais of the openion that this is indicated by the word ādi in preṣaṇādi used by K.[3]

Thus the affix 'ṇic' is employed to denote the activities of inspiration (=act. insp.). 32nd sūtra will ordain this formation as root, or we can say that lakāra will follow ṇic.
Let us form a set of lakāra - affixes as —

T = { Tip, Tas, Jhi, - - - - - - - - mahiṅ }

---

1. hetuḥ - bhavati.
(K. 388)

2 Pa — 391 - 2

3 iV 392 also, N. discussed four possible meaning of the word 'hetumān' and drawn a conclusion in the favour of 'Preṣaṇādi vyapāra' (389)

Any element of this set may be denoted by 't'.
Now, Mathematically —

$$R \frown \underset{\text{act. insp.}}{\text{ṇic}} \frown t.$$

or $$\left(R \frown \underset{\text{act. insp.}}{\text{ṇic}} \right) \in R$$

5. Pleonastic - affixes

Svārthika - affixes — There are some affixes, the addition of which do not alter the meaning of root. Such affixes are called Svārthika affixes. K. and N. explain this phenomenon by stating that the meaning of words is natural and not vācanika[1].

The data belonging to this affix is —

(1) The affix ṇic is added after 'curādi' roots[2].

(2) 'Kaṇḍū' and others take the affix yak[3]. These roots are enumerated in gaṇa-pāṭha and not in the dhātupāṭha. They are roots as well as prātipadikas. When they are treated as roots, only then they take the affix 'yak'[4].

(3) The affix 'āya' is employed after kṛpa, dhūpa, vicchi, paṇi and pani (3.1.28)

---

1. K. 387, N. 387
2. (3.1.25)
3. (3.1.27)
4. K. 400

The root Pani to praise and to bargain is stated with pani to praise. K accepts it as indication of restriction of meaning of Pāṇi i.e. 'to praise only' But Bhaṭṭi also employs the āya after pāṇi to bargain.[6] Ātmanepada is employed after it.[7]

(4) The affix īyaṅ is added after the root ऋ. Indicatory 'ञ्' is for the sake of ātmanepada. It is Sautra-root and means jugupsā.[8]

(6) Root 'Kami' takes affix ṇiṅ. The indicatory letters are for the sake of vṛddhi and ātmanepada.

Affixes ṇic and yak are added in every condition but āya and others are added optionally, when any ārdhadhātuka is to follow. There is no option, when sārvadhātuka affix is to come afterwards

Mathematical Representation —

Let A_γ and S_γ be two sets.

Now,

$S_\gamma = \{ x : x$ is a sārvadhātuka affix $\}$

$A_\gamma = \{ x : x$ is a ārdhadhātuka affix$\}$

---

5 K. 403

6 Pa 403

7 K. 403

8. K. 403

9. K. 404

10. K 406-1 ; 3·1·31

Curādi	ṇic → $_{v}i^{b}$	$_{\sim}$/$_{a_Y}$
Kaṇḍvādi	yak → ya	$_{\sim}$/$_{a_Y}$
gupādi. Pāṇi^{a}/Prāsiṣ	aya → ay^{a}	$_{\sim}$/$_{a_Y}$
ṇī	iyaṅ → iya	$_{\sim}$/$_{a_Y}$
Kami	ṇiṅ → $_{v}i^{a}$	$_{\sim}$/$_{a_Y}$

Another method —

Curā & Rest	gupa & Rest	Pāṇi^{a}/Prāsiṣ	nic ($_{v}i^{b}$) āya → ay^{a}	S/a
Kaṇḍū & Rest	ṇī		iyaṅ → iya	S/a
	Kami		YaK (ya) ṇiṅ → $_{v}i^{a}$	S/a

S a S a

where S_Y = Sārvadhātuka
or A_Y = Ārdhadhātuka.

6. Dhātu - Designating Aphorism.

Aph. 3.1.32 states that the forms of roots of Subanta-words having affixes San and rest as their existing part are designated as roots.

Even an average student of Sans-

-ri Grammar knows that guṇa, vṛddhi and abhyāsa etc. functions take place, there it is also clear that the affixes — sanādi are of two types. Their first group is added after R and second after Px.

The first category contains — san, yaṅ, nic, yak, āya, īyaṅ and ṇiṅ.

The second category contains — Kyac, Kāmyac, Kyaṅ, Kvip, Kvip, Kyaṣ, ṇiṅ and ṇic.

The new designation enables a form of R or Px to be the member of root-set. Let R' and P' be the new forms after performing guṇādi functions.

Then Mathematically — As the new set of roots is a sub-set of R So,

$$\left\{\begin{matrix}(R \longrightarrow R') \frown (san/yan/nic/yak/āya/īyaṅ/ṇiṅ) \\ (Px \longrightarrow P') \frown (Kyac/kāmyac/Kyaṅ/Kvip/Kyaṣ/ṇiṅ/nic)\end{matrix}\right\} \subset \tilde{R}$$

VIKARAṆA—

There are at least 18 Vikaraṇas (Vik.), which are the special kind of affixes as they are added after a root but before the terminating affix - tiṅ (-t). The employment of a Vik. depends sometimes on the nature of t (as-śya tas etc.), sometimes on voice and sometimes on the class of

root (as – śap ; śyan . śnu etc.).
Vik. cli is replaced by other ones.

These phenomena can be expressed in a single form, but that would be somewhat complicated and of less use. So, we shall divide the whole process into four divisions —

(a) General distribution.
(b) The insertion of ām
(c) The substitute of cli
(d) The insertion of yak, śap and others.

(a) **General distribution** — By general distribution we mean the general tendency of the insertion of vik. before lakāras. ~~we~~ We shall not go in detail, at this place. For example we know that before laṭ yak is added to the root in passive and impersonal voice, but in active voice śap, śyan, śnu etc. are added in accordance with the class of root. In the present section, we shall state it as – yak / śap° comes after a root before laṭ. Here, śap° denotes a class of affixes. The addition of different members of this class, however, would be discussed in detail later on.

The general tendencies are —

1. Vik. śa is added before lṛṭ and lṛṅ [1]

---

1. 3.1.33 ; N. 409

2 Vik. Tasi is employed when lut is to follow. The indicatory letter ṣ of Tasi is for the sake of prohibition of the elision of masi around of the root.[3] But the commentator Vāmana regards it for the sake of pronunciation.[4]

3. Sip is diversely the affix of a root when let follows[5]

4 The lit is ārdhadhātuka[6], so Sip° can not be inserted before it, as they can be added only before Sārvadhātuk - affixes[7] Thus, we find the absence of Vik But Vik. ām is added in some cases.[8]

5. Vik - cli is employed before luṅ[9] and is always replaced by other ones. Its indicatory 'c' points the accute accent at the last vowel[10]

6 In all the other cases i.e when laṭ, liṭ, loṭ, laṅ or liṅ follows, then, either the Vik - yak or any one of the Sap class is added This distinction is made on account of voice. Yak is added in passive or impersonal voice, while in active-voice, other affixes are added.[11]

'k' of yak indicates the prohibition of guṇa or vṛddhi[12]

---

2 3.1.33 3. K 2110 - by Jayaditya.
11 N. 410 5 3 1 321 6 3 4.115
7 3.1 67-8 8 3 1 34-9 9 3 1 223
10 N 1131 11 3 1 67 86 12 K 3151

In the reflexive voice i.e., when the object itself is spoken of as agent, then - vik. yak is added.[13] K. is of the opinion that a new vartika is needed for this fact, but Haradatta argues that (3.1.89)[14] indicates that yak is for addition.

'ṅ' of s'ap serves many purposes, as — grave accent, nitya-pratiṣedha, differenciation between s'ap and s'a etc. 's' is for the sake of designation of Sārvadhātuka.

Mathematical - Representation —

Three elements— roots, lakāras and vikaraṇas; are very clear in this process. Besides them, here is the consideration of Sārvadhātuka, ārdhadhātuka, kartṛ-vācya, karm-vācya, bhāva-vācya and karma-kartṛ-vācya. We have to adopt a device to represent all of these.

Pāṇini analyses the Tiṅanta-pada in two parts — root and affix. This 'affix' contains two parts — first of them is the replacement of 'la' (lādesha), and the second part, which is added before la, is well-known as vikaraṇa in the tradition.

Thus,

bhavati ← bhū + s'ap + tip

---

13 K. 4151 ; Pu 3151, AV - 3.5

14 Na dhātu - sva - rūpam yak ; etc.

To get bhavati, we have to apply so many other rules of anubandha-lopa, guṇa, ayādeśa etc. The present process is concerned with the nature of vik. (in this example - s'ap) only.

Let, the symbols be defined —

(1) $\mathcal{S} \in S = \{$ 0, s'ap, s'yan, s'nu, s'a, s'nam, u, s'nā, s'ānac $\}$

— Here, 'S' is a set defined in a tabular form and '$\mathcal{S}$' is its arbitrary element.

(2) Sārvadhātuka $la = la^{+}$

e.g. Sārvadhātuka $laṭ = laṭ^{+}$

" $luṭ = luṭ^{+}$

(3) Ārdhadhātuka $la = lā$

(4) Sārvadhātuka or Ārdhadhātuka $la = la^{\pm}$

(5) Kartṛ vācya $la = la^{\pm K}$

(6) The vācya other than Kartṛ one i.e Karma, bhāva or Karmakartṛ $la = la^{\pm \underline{K}}$

(7) Anunāsika of the root $= n/ṁ$

In root, guṇa or vṛddhi does not take place $= gb \overset{in}{\nrightarrow} ṛ$

or simply as $- gb \nrightarrow ṛ$

Now, the Representation is —

$$ṛ \frown \begin{bmatrix} sya \frown laṭ & \mathcal{S}^{0} \frown laṭ^{+K} & yak \frown laṭ^{+\underline{K}} \\ sya \frown laṅ & \mathcal{S}^{0} \frown laṭ^{+K} & yak\ gb \overset{in}{\nrightarrow} ṛ \frown laṭ^{+\underline{K}} \\ Tas \frown luṭ & \mathcal{S}^{0} \frown laṅ^{+K} & yak\ gb \overset{in}{\nrightarrow} ṛ \frown laṅ^{+\underline{K}} \\ 0n/ṁ \nrightarrow 0 & & \\ sip \frown lel & \mathcal{S}^{0} \frown liṅ^{+K} & yak\ gb \overset{in}{\nrightarrow} ṛ \frown liṅ^{+\underline{K}} \\ o/ām \frown li! & cli \rightarrow sic \frown liṅ & \end{bmatrix}$$

Here, is no symbol of Sārvadhātuka or Vīrya and it, liṭ, leṭ, luṭ and lit. It simply means that no distinction is made on those grounds.

13. Ām Vikaraṇa — No vikaraṇa is employed before liṭ, generally. But there are some roots, after which 'ām' is added The data belonging to this phenomenon is as follows (3.1.35) Affix ām is added after root kās and those which have already taken any other affix These are some roots 'kulumb' and others which take ām affix, still they have not taken any affix. So the form of statement is erroneous one. A vārtika[1] provides the right form as — 'the vik. ām is added after the root kās and those which have more than one vowel'

(1) Pāṇini has used the word āṇa and not ām, so no question arises for the elision of 'm'.[2]
(2) ām is employed after a root which begins with any one of — i, u, ṛ, ḷ, e, o, ai and au, (=ic) if that vowel is pronounced as long one. The roots 'ṛccha' and 'ūṇu' are exceptions (3.1. 36 and KV). This long pronunciation may be due to augment (āgama)?
(3) The same vik is added after daya, aya, and āsa in the same set of conditions (- ssc)

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1. Kasyanekac ... iti vaktavyam ... (KV)
2 K ... N. 418

(4) It is added optionally after 'uṣa', 'vida' and 'jāgṛ' before liṭ. The root is parasmaipadi (jñāne — to know) as it is enumerated between two parasmaipada roots.[4]

Root vida is enumerated at five places as-

1064 vida jñāne. (P),
1171 vida Sattāyām (a),
1432 vida lābhe (a),
1480 vida vicāraṇe (a),
1709 vida cetanā khyānā - nivāseṣu (a)

— Here, the numbers before root are the root numbers of S. K. — 3 and p, a, and, u indicate its nature as parasmaipadi, ātmanepadi and ubhayapadi. Guṇa does not take place in vida as it is enumerated as adanta.[5]

(5) vik. ām is added after the root bhū, hrī, bhṛ and hu before liṭ and functions as if it were also a s'lu (3.1 39) The results of this treatment are the reduplication of first syllable of root and 'i' in place of first vowel.

(6) Though, the aphorism — 'āmaḥ (2.4.81)' does not feature here, but it is very useful to grasp the fact of anuprayoga, fully. āmaḥ means that 'l' is elided after the ām Thus —

ऋ ् ām ् liṭ
= ऋ ् ām ् l (anubandha-lopa)
= ऋ ् ām (āmaḥ)

---

4 Videracuntatua — prati jñānadāni guṇo na bhavati (A 2.20)

(7) After a root having ām, kṛ̃ is added. Thus kṛ̃ is followed by liṭ. (3.1.40) This very aphorism ordains liṭ again.[6] kṛ̃ is prathama-hāra and means — 'kṛ, bhū and às'. Thus case 'às can not be replaced by 'bhu' as in ordinary case happens.[7]

(8) Vik. ām is also added optionally after vida, when liṭ is to follow. kṛ̃, bhū and às are used after them with liṭ (3.1.41)

Mathematical Represenitation —

Some symbols are required to illustrate the process, these are —

The root, having more than one vowel = $^{\mathfrak{R}}$nv.
The root, which begins with the long-ऋ-pronunciated ic vowel = $\overline{\text{ic}}\,\mathfrak{R}$
The form of root after augment = $\mathfrak{R}'$
The first vowel will be replaced by 'i' = $i \leftarrow v^{\mathfrak{R}}$
Reduplication of the first syllable of the root will take place. = $55^{\circ} \leftarrow 5^{\mathfrak{R}}$

---

6. anenaiva sūtreṇa vidhīyamāne liṭi parataḥ kṛ̃ādayaḥ prayujyante iti. liṭ-parataḥ kṛ̃ādayaḥ prayujyante iti vārttikam — (N. 1121)

7. K. 1121

47

Now, the facts can be stated as follows —

(1.)

$$\left[\begin{array}{l} (\text{day}, \text{aya}, \bar{\text{a}}\text{s}, \text{K}\bar{\text{a}}\text{s}) \widehat{\ } \quad (\text{u}\dot{\text{s}}\text{a}, \text{vida}^{\text{n}}, \text{j}\bar{\text{i}}\text{g}?) \widehat{\ } \\ \qquad\qquad\qquad\qquad\qquad\qquad 8 \rightarrow 9 \\ \overset{R_{nv}}{\text{Tc}} \left[\text{R}' - (\text{r}\dot{\text{c}}\text{cha} + \bar{\text{u}}\text{rn}\text{u}) \right] (\text{bh}\bar{\text{i}}, \text{hv}\bar{\text{i}}, \text{bh}\text{r}, \text{hu}) \widehat{\ } \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{ss} \leftarrow \text{s}^{\circ} \end{array} \right] \widehat{\ } \bar{\text{a}}\text{m} \widehat{\ } \underset{\downarrow \circ}{\text{l.t}} \widehat{\ } \underset{\parallel (\text{M/bh}\bar{\text{a}}\text{s})}{\text{K}\text{r}\tilde{\text{n}} \text{ 'lit}}$$

2. $\text{Vida} \widehat{\ } \bar{\text{a}}\text{m} \widehat{\ } (\text{lo}\text{t} \rightarrow \text{o}) \widehat{\ } \text{K}\text{r}\tilde{\text{n}} \widehat{\ } \text{lo}\text{t}$.

C. **Cladesa** — Though, the aphorism 3.1.43 ordains the addition of Vik. 'cli' before luṅ. But, there is no root where it can be found, as it is re-placed by other ones always.

The data of this replacement is given below —

1. (a) Vik. 'sic' in the replacement of 'cli'.[1] This state-ment gives the effect that 'sic' can be found after every root, but it is not so. This is a gen-eral rule and special-rules would be stated, afterwards. 'i' of 'sic' is for the sake of pronunci-ation and 'c' for the acute accent on the last s-yllable.[2]

(b) Sic is added optionally after roots spṛs'a, mṛs'a, Kṛṣa, Tṛpa and dṛpa.[3] Basically, first three roots take the Kṣa and remaining two the 'an'. The present vārtika makes them optional, i.e. —

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1. 3.1.44
2. K. 2130, Pa 2130
3. Kv. 2132 and Av. — 1

(Spṛśa , mṛśa , kṛṣa) ⌢ Sic (⌢ Ksa) ⌢ luṅ
(tipa , dṛpa) ⌢ Sic (⌢ aṅ) ⌢ luṅ [4]

2.

(a) Vik. 'Ksa' replaces 'cli', if the root ends with any letter of S'al pratyāhāra, whose penultimate letter is any one of the vowels— ऌ, ऊ, ॠ and ॡ and does not take the augment 'it'. Indicatory K is for the sake of prohibition of guṇa.[5]

→ ॠ it (= --°(iK) (S'al)] ⌢ Ksa ⌢ luṅ

(b) Root 'S'liṣ' takes Vik. 'Ksa' when it means to-embrace. In other meanings it takes aṅ as it is enumerated in Puṣādi set of roots.[6]

Mathematically,

S'liṣ / embrace ⌢ Ksa ⌢ luṅ
S'liṣ / other meanings ⌢ aṅ ⌢ luṅ

(c) Vik. 'Ksa' is not added after dṛṣ', though the root fulfills all the conditions laid down by 46th aphorism[7]. Aṅ and caṅ are employed optionally.[8]

dṛṣ' ⌢ aṅ / caṅ [↑ Ksa] ⌢ luṭ.

---

[4] By now, the reader must be able to understand and even to use himself the facts written in mathematical language so we shall use them frequently.

5 Pa 3133. 6. 3 1 36 ; Pa 3134, N. 3135 35

7 3.1.45 8 3.1.47

(3)

a. — Vik 'cari' is added after the roots — s'ri dru sru, and those which have taken 'ni' affix, when kartṛ denoting the subject, is to follow. The 'n' of cari indicates the prohibition of guṇa or Vṛddhi.[9] Root 'kami' must be enumerated here mathematically.[10]

$[s'ri + dru + sru + (ve + ṇi) + kami] \frown cari \frown luṅ^K$

$g^b \rightarrow \text{ṛ}$

b. — In active voice (i.e. when luṅ denotes subject) —, dhet and s'vi take cari, optionally. The optionals are sic and cari, respectively. By the force of rule — 2.4.78 sic may be elided. Thus, we have three forms of dhet and two of s'vi.

Mathematically —

$dhet \frown (cari / sic / sic \rightarrow o] \frown luṅ^K$

$s'vi \frown (cari / aṅ) \frown luṅ^K$

c. — We are not considering the uses related to vedic literature simply because it has frequent options. There is every possibility of other forms of the words, even if we try to give full details.

(4)

a. — Vik. aṅ is added after the roots asu, vac and ikṣvṛ in active voice.[11] Here, vac is either root or the replacement of root bruṅ. Root — asu can take this affix by the force of the rule 3.1.55 as it is one of the Parāḍi roots. So its enumeration in 2nd rule is for the sake of ātmanepada

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9 3.1.48

10 KV AV I II 3.1.32

(cisu , vac , khyā) ^ ani ^ luṅ [k]

b. — Root – lip, sic and hu always take this vik in parasmaipada luṅ[12] (in active voice)[13]. In atmane-pada the addition is optional and there is no restriction of voice.[14]
Mathematically —

(lip , sic , hu) ^ ani ^ luṅ [pk]
and (lip , sic , hu) ^ ani ^ luṅ a

c. — Before parasmaipada luṅ, this vik is added after puṣādi-set, dyutādi-set and roots with indicatory ḷ.[15] Puṣādi is a sub-set of three sets viz. bhū, kṛiyaṁ and dev[illegible] class out of these three subsets, last one is meant here.[16] Mathematically —

[Puṣ° + dyut° + ṛ (ḷ → o)] ^ ani ^ luṅ [p]
or, [Puṣ° (⊂ div°) + dyut° + ṛ (ḷ → o)] ^ ani ^ luṅ [p]

d — This vik. is added only after sṛ, sās and ṛ[17]. The seperate enumeration indicates the addition in both padas. (K. 4442)
Mathematically —

(sṛ + sās + ṛ) ^ ani ^ luṅ

---

12 3.1.53
13 As I 370 (Aṣṭādhyāyī - by S ([illegible] Vasu)
14. 3.1.54 , As I 370
15 3.1.55
16 K [illegible] N 4441
17 3.1.56

e. — The roots, having indicatory ir, take this vik optionally in parasmaipada (3.1.57). sic is added in ātmanepada.

Mathematically —

$$\begin{cases} \text{ṛ (ir} \rightarrow 0) \frown \text{an} \frown \text{luṅ}^{b} \\ \text{ṛ (ir} \rightarrow 0) \frown \text{sic} \frown \text{luṅ}^{a} \end{cases}$$

f. — This vik. is added optionally after the roots- jṛ, stambh, mruc, mluc, gruc, gluc, gluñc and śṛi in parasmaipada.[18]

g. — There are some vedic uses of this vik. (3.1.59)

(5)

a. — Affix cin is the substitution of .ci after the root 'pad' when ta (of ātmanepada) is to follows, before other termination sic replaces ci.[19]

Mathematically,

pad ⌢ cin ⌢ ta

pad ⌢ sic ⌢ luṅ (-ta)

b. — Roots, dīp, jan, budh, pūr, tāy and pyāy take cin optionally in the above mentioned conditions.[20]

Mathematically,

(dip, jan, budh, pūr, tāy, pyāy) ⌢ cin ⌢ ta

c. — cin is added optionally after a root ending with vowel in reflexive voice i.e when object-

18 . 1.68 19. 3.1.60 20 3.1.61

is spoken of as the agent.[21] If we represent the Karmakartṛ as – KK'.

Mathematically,

$$\sqrt{} \frown \text{cañ} \frown \text{ta}^{KK'} \ (\leftarrow \text{luṅ})$$

(d) The root 'duh' also takes the vik. Ciṇ optionally in reflexive use before ta – termination.[22]

$$\text{duh} \frown \text{cañ} \frown \text{ta}^{KK'} \ (\leftarrow \text{luṅ})$$

(e) The root 'rudh' does not take ciṇ in reflexive use,[23] but takes it in other ones.

$$\text{rudh} \uparrow \text{ciṇ} \frown \text{ta}^{KK'}$$

(f) ciṇ is not added after a root 'tap' when it is used in the sense of remorse (anutāpa) in the reflexive use.[24] This prohibition is applicable in passive and impersonal voices too.

Mathematically —

$$\text{Tap/remorse} \uparrow \text{ciṇ} \frown \text{ta}^{KK'+K'+b}$$

Here, K' = Karm

and b = bhāva.

(g) 'ciṇ' is the substitute of 'cli' when 'ta' denoting action or object follows. (— after any root)

$$\sqrt{} \frown \text{ciṇ} \frown \text{ta}^{K'+b}$$

---

21. 3.1.62
22. 3.1.63
23. 3.1.64
24. 3.1.65

Mathematically Representation —

(1) Kṣa —

$$\left[\begin{array}{l} \{(\underset{\rightarrow}{R}\ \underset{il}{=}\ (- + iK - s'n\dot{l}) + \underset{\text{samprasāraṇa}}{Sl\dot{s}/ - el\dot{s}s'}\}^{\frown} \\ \rightarrow g \\ (Sl\dot{s}s' + mr\dot{s}' + Kr\dot{s})^{\frown} sic \\ \rightarrow g \end{array}\right]^{\frown}[Kṣa \rightarrow ṣ]^{\frown}[luṅ]$$

(2) Cañ — $\left[\begin{array}{l} \{(ṛ + ṇi) + s'vi + dru + sru + Kam\}^{\frown} \\ dhe\dot{t}^{\frown} sic^{\frown} / sic \rightarrow o \\ s'vi^{\frown} añ \end{array}\right]^{\frown}[cañ \rightarrow a]^{\frown}[luṅ]$

$\rightarrow$ gb
abhyāsakārya

(3) añ — Thus mathematically —

(i) (asu^{a}, vac (ṛ / = bruñ), khyā)$^{\frown}$ añ $^{\frown}$ luṅ^{K}

Here, = means ādeśa

~~(ii) (lip, sic, hu)$^{\frown}$ / añ $^{\frown}$ luṅ^{p}~~

(ii) [puṣ$^{\circ}$ ⊂ div$^{\circ}$, dyu$^{\circ}$, R (ḷ $\rightarrow$ o)]$^{\frown}$ añ $^{\frown}$ luṅ^{p}

(iii) (sṛ, Sl's ṛ)$^{\frown}$ añ $^{\frown}$ luṅ

(iv) R (iṛ $\rightarrow$ o)$^{\frown}$ añ / sic luṅ p/a

(v) (Jṛ, Stambh, mruc, mluc, gruc, gluc, glauñc, svi)$^{\frown}$ añ $^{\frown}$ luṅ

(vi) (lip, sic, hu)$^{\frown}$ añ $^{\frown}$ luṅ^{pK}

(vii) (lip, sic, hu)$^{\frown}$ añ $^{\frown}$ luṅ

(4) Cin —

(i) pad $^{\frown}$ cin / sic $^{\frown}$ ta (luṅ) / (luṅ — ta)

(ii) (dīp, jan, budh, puṣ, tan, kṛṣ)$^{\frown}$ cin $^{\frown}$ ta (luṅ)

(iii) duh + R^{V} ∩ cin ⌢ ta ^{KK} [KK = karma karta]
(i.e Reflexive voice)

(iv) duh ∩ cin ∩ ta ^{KK}

(v) Tap/remorse ↑ cin ⌢ ta ^{KK} + K' + b.

(where K' = karma
b - bhāva)

D. Yak and S'ap & the Rest. —

In remaining lak-āras either vik. Yak or s'ap and others are used The data, belonging to this process is as follows —

(1) The vik. Yak is added to a root when a sārvadhātuka expressing either action itself or the object follows.[1] 'K' of Yak is for the sake of the prohibition of guṇa and vṛddhi

Yak is also added in the reflexive use ie when object is regarded as subject. Jayad-itya wants a seperate enumeration for this, as it is not included in the rule given by Pāṇini. Av-6 explains this phenomenon of language by breaking the (3.1 67 ^) aphorism into two parts. But Haradatta says that this is understood by the prohibition of Yak in reflexive use by the rule — 3.1.89 [2]

---

1 A 3.1 67

2 K 1161, AV. 3-6, Pa - 1151

2

(a) The affix śap comes after a root when a sārvadhātuka, expressing the agent follows.[3] 'ś' and 'p' of śap indicate guṇa accent and designation of sārvadhātuka, respectively.

(b) The affix śyan comes after a root of divādi class within the conditions mentioned in (Rā – ṣ)[4] Indicatory letters 'ś' and 'n' are for the sake of sārvadhātuka and accent (the acute accent falls on the root and not on the affix).[5]

This affix is employed optionally after the roots bhrāś, bhlāś, bhram, kram, klam, tras, truṭ and laṣ in the same conditions.[6] These roots are of div- class.

Root yas takes this affix, optionally ~~after~~ when it is used without prepasition. But the prepasition sam is exception he this affix is used optionally after (sam + yas). As this root belongs to the set div°, so in other cases śyan is added.[7]

(c) Affix śnu is added after a root belonging to svadi class, in the same set of conditions. ie in (Rā – ṣ).

Śnu takes the guṇa of ś simultaneously with addition of this affix, though this root belongs to bhū class.[8]

---

3 3.1 68

4 3.1 69

5 6.1 197

6. 3.1 70

7 3.1.71 2

8 3.1 73 4

There are two roots of bhū class which take this affix optionally in (Ra ⌒ — ⌒ s), They are 'aks' - to pervade or obtain, and 'taks' — to hone or hew. In the case of taks the above meaning is to be followed strictly.[9]

(d) Tud- class takes the affix 'śa' in (R ⌒ — ⌒ s)[10]

(e) Rudh class takes the affix s'nam in (R ⌒ — ⌒ s). The indicatory letter 'm' is for the sake of the place of the addition of the remaining part of the affix. (= ṇa) which is, obviously, after the last vowel of the root.[11]

(f) Affix 'u' is added after tanu class and kṛ in (Ra ⌒ — ⌒ s). The root kṛ belongs to tanu class. Its seperate enumeration here indicates the non- application of other functions of tanū class to it.[12] Roots 'dhinvi' and kṛṇvi of bhū class, also take affix 'u' in (Ra ⌒ — ⌒ s) and 'a' replaces their final vowel ie. 'i'.[13]

(g) The affix s'nā comes after the roots of krī ñ — class in (Ra ⌒ — ⌒ s).[14]

The roots stambh, stumbh, skambh and skuñ takes affix s'nā as well as s'nu.[15] The first four roots, enumerated here, are found in aphorisms and not in dhātupāṭha.[16]

---

9. 3. 1. 75 6
10 3. 1 77
11 3 1 78 (R. 2146 A 3 1 71)
12 3 1 79
13. 9. 1. 80
14 3. 1. 81
15 3 1 82
16. K 2159

(h) The affix s'ānac is the substitute of s'nā after a root ending with consonant, when affix hi (1st, II[nd] person. Sing. number) follows.[17]

$$R_c\hat{}\,|\ s'nā \longrightarrow s'ānac\ |\hat{}\ hi$$

(i) Next two aphorisms deal with vedic language. We are not considering them here, as a good deal of changes from classical language are very common here.

(j) There are some classes of roots which are important for consideration. They are ada, hu, and cur class. After ada° the vik. is elided.[18] The vik. is also elided after hu°, but this elision is designated as s'lu.[19] Cur° takes nic affix before taking s'ap.[20]

Mathematical Representation —

$$\text{Let } S = \{ \text{laṭ}, \text{loṭ}, \text{luṅ}, \text{liṅ}, \text{Ās'} \longrightarrow o \}$$

$$s^{b+K+KK^{\pm}} = s \text{ expressing bhāva, Karma or Karma-Kartā.}$$

$$s^{aq} = s \text{ expressing agent.}$$

Here, we find three categories of addition.

(i) The addition of Yak

(ii) The addition of śap° (A) Regular Roots.

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17 3. 1. 83

18. 2. 4 °12

19 2 4 °15

20. 3 1. 86

(iii) The addition of śap° (B) The irregular Roots.

By regular roots, we mean those roots, which take vik. of their own class.

Now Representations are —

(i) $R \frown (Yak \rightarrow Ya) \frown \acute{s}^{b} / K / KK'$

$\not\rightarrow gb.$

(ii) Regular roots – (with only three exceptions)

$$\begin{bmatrix} bh\bar{u}^{\circ} \\ ac\dot{l}a^{\circ} \\ hu^{\circ} \\ div^{\circ} \\ su^{\circ} + (Sra \rightarrow Sr) \\ tud^{\circ} \\ \acute{s}udh^{\circ} \\ tan^{\circ} + (dhinvi + Kr\dot{n}vi) \\ \qquad\qquad \downarrow a \qquad \downarrow a \\ kr\tilde{i}\tilde{n}\tilde{a}\ Kr\bar{i}\tilde{n}^{\circ} \\ cur^{\circ} \end{bmatrix} \frown \begin{bmatrix} \acute{s}ap \\ 0 \\ 0 = \acute{s}lu \\ \acute{s}yan \\ \acute{s}nu \\ \acute{s}a \\ \acute{s}nam \\ u \\ \acute{s}n\bar{a} \\ nic + \acute{s}ap \end{bmatrix} \frown |\acute{s}|$$

(iii) Ir-regular forms —

Let,

R_1 = truṭ

R_2 = (bhrāś + bhlāś + bhramu + kramu, + klmu + triṣi + laṣ + (o+yaṣ), + (sam + yaṣ)]

R_3 = (akṣa + takṣa / to bore or hew)

R_4 = (Stambhu , Stambhu , skambhu , skumbhu , skuñ)

A = (śa , śyan , śap , śnu , śna)

R = (R_1 , R_2 , R_3 , R_4)

Now, the ir-regular forms obey the following rule —

$$R_j \frown A_{i/i+1} \frown s^{ag}$$

8. The Reflexive Voice —

(a) Karmavat-Karmaṇā tulya Kriyaḥ (3.1.87)

Jayāditya explains it as — If the action of the agent is like the action within the object, that Agent is treated as Karmakartā. The term 'Karma', used here is technical one.[1] This is a Kāryāt-idesa.[2] For a single root the agent behaves like an agent as well as object.[3] The word 'Kriyā' includes 'Kriya' and 'bhāva'. S.C. Vasu, in nut-shell translates the above aphorism as — 'The 'la' denoting the agent, when the action affecting the agent is similar to the action which affects the object, is treated as if it were a 'la' denoting the object.[5] This similarity of action must be with object and not with instrument or location.[6]

---

1. Pa 1166
2. Pa. 1168-9 ; N 1168.
3. Ekasminneva dhatau Karmakartri bhave bhavati. (N 1179)
4. N. 474 ; Pa 414
5. 380
6. ibid

(b) The agent of the root 'tapas' - to heat, becomes similar to the object only when object is the word 'tapas' itself.[7]

(c) In the reflexive voice 'yak' and 'cin' are not employed after roots — duh, snu and nam. (3.1.89). This prohibition must be extended to the roots those take the affix ṇi (nic. or nin), to the roots 'granthi', 'granthi', bru and to the intransitive roots, which take the ātmanepada affix.[8]

(d) In the reflexive voice, roots 'kus' and 'rañj' take the affixes śyan and that of parasmaipada, according to the opinion of Eastern Grammarians.[9] The word 'prācām' (Easterne-rammarians) shows that it is an option rule.[10] However, it is a restricted option (vyavasthita-vibhāṣā) and is not applicable when śya & R are added.[11]

(e) The aims of this atideśa are — yak, ātmanepada, cin and cin-vad-bhāva.[12]

---

7. 3.1.88
8. Kv. 3.1.8
9. 3.90
10. K. 481
11. " 481
12. Yagatmanepada cincinvad bhāvah prayojanam (K. 3.1.89)

Mathematical Representation —

Let, O = object

O_{ab} = action or bhāva of object

A_{ab} = action or bhāva of agent

$int.^{a}$ = The intransitive root having Ātmane-pada affix.

$\sim$ = Similar

$\Rightarrow$ = implies.

Above facts can be stated as —

$$A_{ab} \sim O_{ab} \Rightarrow \left| \begin{array}{l} \text{ऋ} \\ \text{tapas } (o = \text{tapas}) \\ \{\text{duh, snu, nam, śrant-hi, granthi, bru, ऋतॄ इत्}^{a}\} \\ \text{kuṣ, ranj} \end{array} \right| \cap \left| \begin{array}{l} \text{Yak, cin, cinṇadhātau} \\ \text{" " "} \\ \text{x x "} \\ \text{Yak/śyan cin "} \end{array} \right| \cap \left| \begin{array}{l} la^{a} \\ la^{a} \\ la^{a} \\ la^{a/p} \end{array} \right|$$

$$\Downarrow$$

$$A_g = KK'$$

9. Upapadam —

Tatropapadaṃ Saptamīstham (3.1.92)

A word ending in the 7th case affix used in a aphorism in which dhātu (3.1.91) is understood, is designated as - Upapadam (K 485)

It is a well-known fact that the upapadas are used before verbal forms. Upapada and kṛdanta forms are united (becomes Samasta pada) before the addition of sup-affixes.

Mathematically, we can state —

Let, U -- upapadam

Then,

$U = \{ X : X$ is a word ending in 7th case affix and is used in a rule containing the anuvṛtti of dhātu (3.1.91)$\}$

$\omega = \{ U (\alpha \frown A_1) \} \frown A_2$

where, ω = word

A_1 = Affix tiṅ or Kṛt

A_2 = Affix Sup.

10. Kṛt (Affixes) Designation —

Kṛdatiṅ (3.1.93)

The affixes minus the 'tiṅ' ones, described by the rules in which word – dhātu is understood, are called Kṛt.

Mathematically,

$Kṛt = \{ X : X$ is an affix ordained by a rule containing the anuvṛtti of dhātu and $X \in \{tiṅ\} \}$

11 Debarment of Affixes — The affixes within present portion (i.e. in which the word dhatu is understood from (3.1.91)) are of two types Some of them are general affixes and others special ones, which are applicable after some definite roots. So these definite roots take general affix also. The sutras are :-

Vā (a) sasicha (a) stasijasi (3.1.94)

In this portion of Sūtras in which there is a reference of verbal roots, an affix which is calculated to debar a general one, not being of the same form optionally debars it. But not so in the case of feminine affixes.[1]

If two affixes are of same form, the preemption will not be applicable and special affix would debar the general one.[2]

In the case of feminine affixes, a special affix will debar a general affix, even though they are dis-similar in form.[3]

The similarity or dis similarity is calculated on account of remaining form of the affix dis similar indicatory letter donot make an affix dis-similar.[4]

Actually, here the word 'Strīyām' stands for the same word used in "Strīyām ktin" (3.3.94) and means that that thus optional debarment is not applicable to the affixes ordained by the rules in which word Strīyām (3 3 94) is understood.[5]

Mathematically —

$$|R| \curvearrowright \begin{cases} (a=I+S)^{\times} & (a=I+ch) \\ (a=I+S)^{\times} & (a\neq I+ds)^{\times} \end{cases} \longrightarrow \text{feminine (gen) affixes} \longrightarrow \text{feminine (gen) affixes} \quad (3.3.94)$$

(gen) affixes = the matrix of general affixes

---

1 Ast. 382 3 — 4 K 493 4

2 ibid 383 — 5 K 491 2 Pa. 492

3 ibid

feminine = word Striyam (3.3.94) is understood (3.3.94).

I + S = Indicatory letters + similar form

I + ds = Indicatory letters + dissimilar form

X = is declared

12. Kṛtya - designation.

Kṛtyāḥ (prāṅ ṇvuloḥ) (3.1.95)

From this aphorism, as far as 'ṇvula' is stated in 3.1.133, all affixes are designated as 'Kṛtya'.

These affixes are — tavyat, tavya, anīyar, kelimar, yat, kyap and ṇyat.

Mathematically, we can say —

(i) Tabular - method:

Kṛtya = { tavya, tavyat, anīyar, kelimar, yat, kyap, ṇyat }

(ii) Property - method:

Kṛtya = { x : x is a Kṛt affix ordained before rule 3.1.133 (ṇvul - tṛcau) }

(iii) In Interval form:

$$\text{Kṛtya} = [\text{affix}]_{3.1.96}^{3.1.133}$$

13. Kṛtya - Affixes —

Pāṇini gives general rules first and then the special rules here

-rally these affixes express either action or object.[1]
The data belonging to this process is as follows —

(a) The affixes 'tavya', 'tavyat' and 'anīyar' are added after the roots[2]. Last 't' of tavyat and 'a' of anī-yar is for the sake of the accent. i.e. 'tavya' has circumflex and anīyar has acute accent on the mi-ddle vowel.[3]

'Tavyat' is added after the root 'vas' ex-pressing agent and 'nic' is added simultaneously.[4]

The affix 'kelimar' should be enumerate-d here.[5] This affix is added in reflexive voice[6]. 'k' of 'kelimar' is for the prohibition of guṇa and vṛddhi a-nd 'a' indicates the accent. (Pa 495)

(b) The affix 'yat' comes after a root that ends with a vowel but ā (3.1.97 ; 3.1.124). The roots 'tak', 'śās', 'cat', 'yaj' and 'jan' should be enumerated in this list of roots. (KV. 497)

Root 'han' takes it optionally (nyat) a-nd 'vadh' is substitute of it before 'yat'.[7]

Affix 'yat' is added after those roots which have a labial letters as final one, and short 'a' as penultimate one.[8]

---

1. S. 4. 70
2. 3 1 96
3. K 495, 6.1.185, 6.1.217
4. KV, 495, N. declares this vā-chika but agrees with this phenomenon of the language.
5. KV – 2
6. KV – 497
7. KV – 497
8. 3.1.98

'Yat' is also added after 'śak' (to be able) and 'śah' (to bear).[9] Gad, Mad, Cara and Yama take this affix when no preposition is added.[10] However, root 'car' takes it with preposition ān, but the sense is one different from preceptor."[11]

The five rules (3.1.101 – 105) give the list of words which are formed with the addition of this affix plus ir-regular operation. 'Ete śabdāḥ nipatyante' – says Jayāditya.

(C). Kyap as well as Yat is added after the root vad, governing a case inflected word as its upapada and not having preposition annexed to it.[12]

'Bhū' takes 'Kyap' in SSC (Upapada and preposition) to denote condition.[13] (bhāva)

Root 'han' takes this affix in SSC (Upapada, preposition and condition (bhāva)] and 't' is the substitution of final of 'han' simultaneously. It is always used in feminie gender.[14]

Root – 'iṇ', 'stu', 'śas', 'vṛñ', dṛ and juṣ take this affix (3.1.109). This aphorism cites the word Kyap again, even though it was available by anuvṛtti. This shows that no other affix is applicable here, but in this rule, above cited three conditions are not applicable. 'vṛñ', stands the root Vṛ.[16]

---

9. 3.1.99
10. 3.1.100
11. Kv 499
12. 3.1.106
13. 3.1.107
14. 3.1.108 ; Pa. 603
15. K. 604
16. K 605

Roots — śās, duh and duh take this affix optionally (ञ्यत्). Root 'añj' (to anoint) takes this affix when preceded by ava and used as appellative.[17]

After the roots having short -ṛ- or penultimate letter, kyap is employed with the exception of the roots kḷp (to be able) and cṛt (to hurt). Sṛj is the exception (ञ्यत्) when preceded by sam, ava or word pāṇi.[18]

Kyap is employed after the root 'khan' and 'ī' is the substitute for the final of the root.[19] This affix kyap is employed after the root 'bhṛñ' when not used as a name. The addition is optional (ञ्यत्) with the preposition sam.[20]

It is added optionally (ञ्यत्) after the root 'mṛj' (3.1.113)

The root 'grah' takes it when it means — word, dependent, outsider or partisan.[21] kṛ and vṛṣ take it optionally.[22]

There are some words where affix kyap and other irregular operations are found. Such words are enumerated in the rules — from 3.1.114 – 123. The nature of this description resembles that of dictionary.

(d) The affix ञ्यत् is employed after a root ending either with ṛ or a consonant.[23] This affix is added ञ्

---

17 KV. 505

18 KV (S) 506, 3.1.110

19. 3.1 111

20. 3.1 112 ; KV

21. 3.1.119

22 3.1.110

23. 3.1 124

-ter a root ending with 'u' (short or long), when the sense is of necessity.

This affix - ṇyat is added after the roots- āsu, yu, vap, rap, lap, trap and cam. The word ca in rule 3.1.126 indicates the enumeration of uncited roots in the aphorism- for example - 'dabh'[26]. The next six aphorisms enumerate irregular words with particular meanings. These words are formed with the affix ṇyat

Mathematical Representation — It is always useful to understand the general tendency first and then particular cases. We will present our data in this light

A. General forms.

R	tavya, tavyaˈ(t), anīya(r)	bK
Vas + ṇic	tavya	a
R → gb	kelīma (ṣ)	KK'
R = °ap + R (V - ṣ)	Yat	bk
$R_{ṣ} + R_c - R_u$ / necessily	ṇyaṭ	bK
{R = ṇic - (K!p+cvi)}	Kyap	bk

Here, ° = ucārṇa

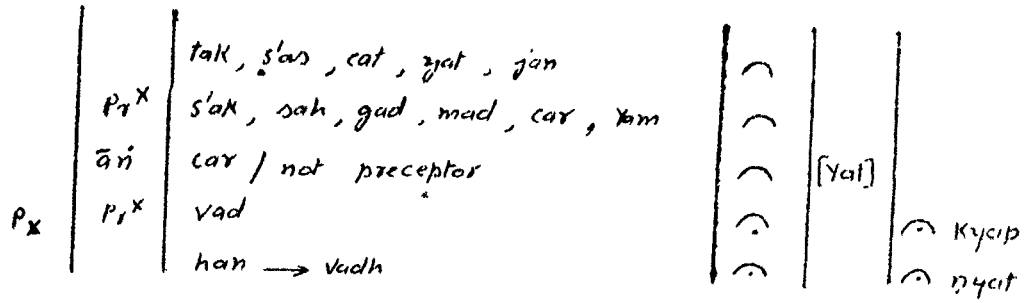
! - svarita

---

26 k 620

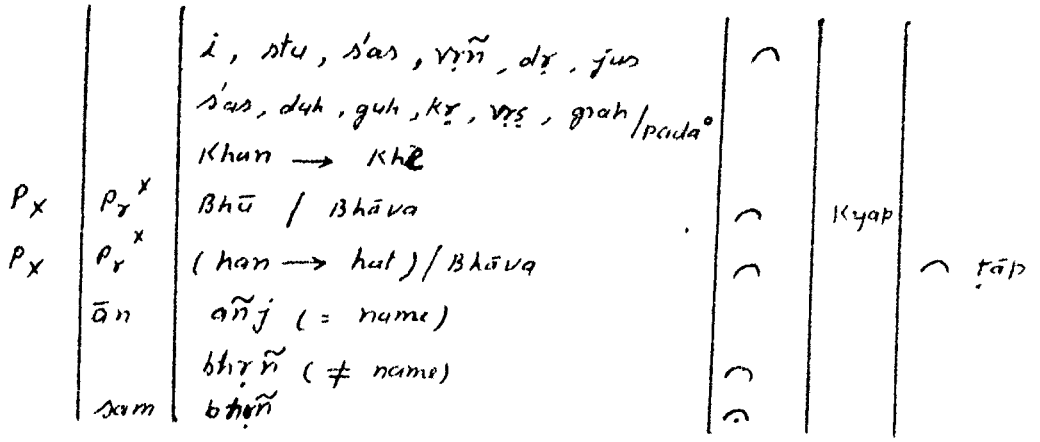
B. Particular Cases.

(i) Yat_



Here — Pr = preposition

(ii) Kyap_



Here — Pr = preposition

(iii) Nyat_

(ā su, yu, vap, vap, dup, bhup, cam + x) ṇyat

14. Kṛt - Affixes.

(a) General — Ṇvul - tṛcau (3.1.133)

Affixes 'ṇvul' and 'tṛc' are added after a root expressing the agent'. Indicatory letter 'ṇ' is for Vṛddhi. 'c' is not the indicatory letter, it is attached to the affix so that simple tṛ may denote both 'tṛn' and tṛc.[2]

Affix 'ac' is also added after all the roots. A verse quoted by Jayaditya is —

aj vidhiḥ sarva - dhātubhyaḥ pathyante ca pacādayaḥ |
an vādhanārtharneuaṃ Syut siddhyanti s'uapacādayaḥ ||
— K. 529.

The word ādi in pacādi is in the sense of — 'a kind of'.[3] The rule given by Pāṇini is —

Nandi grahi pacādibhyaḥ ~~lyuṇi~~yacaḥ (3.1.134)

Here, we are concerned with Pacādi class only. These roots take affix ac. Jayāditya enumerates them as —

Paca, vaca, vap, vada, cula, s'ala, Tapa, Pata, nadaṭ, bhaṣaṭ, van, garaṭ, Plavaṭ, caraṭ, Tarat, Corat, gāhaṭ, Jana, mana, Kṣara, Kṣana, Sūdaṭ, dīvaṭ, modaṭ, Sena, meṇa, kapa, nadhaṭ, narta, vraṇa, dars'a, daṃs'a, stambha, Jāgarabharā and s'uapaca.

This set is infinite set.[4] Here we find that 'ṭ' is all

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1 399

2. 399, K. 525

3. ādi - s'abdaḥ prakare (N. 530)

4 akṛti gaṇa — K. 529

-ached to some roots. This is an indication that in feminine genders, affix - nīp is added after such words.

We can say, in short, that 'ac' is a general affix. The enumeration of them is to show that affix 'an' can not be employed to the listed roots.

Mathematical Representation —

$R \rightarrow b$	⌢	ṇvul	
$R \rightarrow g$	⌢	tṛc	
$R + Pac^{\circ} \rightarrow g$	⌢	ac	(aṇ + ↑aṇ)

15. Kṛt - Affixes: Non - General cases.

Nandi - grahi - pacādibhyo lyuṇinyacaḥ (3.1.134) Affixes lyu, ṇini and ac are added after the sets of nandādi, grahādi and pacādi respectively.[1] No class of these three is taken from one place of dhātupāṭha. The words formed by these roots and affixes are collected from gaṇapāṭha.[2] We shall discuss these classes, sepesately.

(a) Nanda Class,

This class takes the affix lyu (= ana). Roots nand, vaṣṭi, mṛḍ, dūṣi, sidhi, vardhi, śobhi and jṛi take this affix along with ac affix to form the names[3] śak, lap and dam take it to form names[4]. The other roots

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1. 399
2. K 525
3. Kv 526, Jy (1) 400
4. Kv 626

of this class are - tṛ̥pa , śram , klam , śam + kṛand , śam + kaṣ , śam + haṣ , jan + ard° , yu , madhu + śād , vi + bhāṣ , dru[6] , vella + vi + nāś , kul + dumra and śatru + dam.

(b) Grah - class.

This class takes affix ṇini. The roots in this class are grah , ut + sah , ut + vas , ut + bhās , sthā , m- antra , sam + marda , ni (+ śrak'ṇa , s'ru , vas , vah - s'ā)[7] a (+ yac , vi + ā + hṛ , sam + vi + ā + hṛ , vraj , vad , vas)[8] a (R_v)[9] if the agent is non conscious being, vi + (s'a , ṣa) when meaning - place , abhi + bhū , apa + rādha , upa + rodha , pari + bhū + nic , pari + bhū.[10]

(c) Pacādi - class.

As stated above, ac is a general affix. But in particular cases it is debared by ka , s'a , and ṇa. So it is being considered here also.

(i) Ka - affix -

After a root ending in a consonant and having i, u or ṛ (long or short) as its penultimate letter and after the roots - jñā , pṛī and kṛ comes this affix.[11] (and not aC)

This affix is also used after a root , ending in ā and used along with a preposition.[12] It is also added with 'grah' when agent is observed dexter - ii house.[13]

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5. karmaṇyaṇi prāpte svamātra- -ārthi karmaṇyup pade dṛśyarthaṃ. Pa - 6 26

6 it anubandha 'ṛ' (K. 6 26)

7 Kv 1 621

8 Kv 2 621

9. Kv. 3 328

10. K 528

11. 3 1 135

12. 3 1. 136

13. 3 1 144

Asi 4041

(ii) Śa - affix —

Affix śa is added after the roots pṛ, ghrā, dhmā, dhaṭ and dhṛṣ when there is a prefix-ation along with these roots.[14] Some commentators do not read the word 'upasarga' into this aphorism[15]. But as the next rule cites the pada - 'anupasargāt', this explanation can not be accepted. 'Ś' of 'śa' is for the sake of the designation - 'śāruadhātuk.' Prohibition mast be stated by the root 'ghrā', when the word to be formed is a name.[16] (ka is added)

This affix is also employed after the roots limp, vind; (dhṛ, pṛ, vid, ut+ej, śāṭ, sah) + nic, when used without a preposition.[17] The root 'śāṭ' is a sautra root.[18] (dhātupāṭha). Nit limp, and go + vind, however take this affix.[19]

Roots dā and dhā take this affix optionally (ना ṇa) when used without a preposition.[20]

(iii) Ṇa - affix —

The affix 'ṇa' is employed optionally (ना ac) after the roots beginning with 'jval' and ending with 'kas', when used without a preposition. ie ति [jval ..... kas].[21] The word 'iti' is used in the sense of ādi in 140 rule - 3.1.140.[22]

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14 3.1.137
15 K. 630
16 KV
17 3.1.138
18 K. 631
19. KV (ś)
20. 3.1.139
K. 632
21 3.1.140
K. 633
22. K. 533

This affix is also added after the following roots - S'yai, Rā, vyadh, āsru, saṃsru, atiṅ, aruṣā, oṣahṛ, dih, s'liṣ and s'vas.[23] This seperate enumeration of these roots indicates that no other affix can be added after these roots.[24]

The root 'Ton' takes this affix, when used with preposition, the option is not applied here.[25]

This is also employed after 'du' and 'ni' roots when used without preposition.[26] Root 'grah' takes this affix optionally (ā ac)[27]. This option is well-arranged (vyavasthito - vibhāṣā). When meaning grāh (water animal), it is added and when meaning heavily laminarces, it's not employed.[28] Bhū also takes this affix.[29]

Mathematical - Representation —

$$\text{names} \left| \begin{array}{l} (\text{nand}, \text{vāṅ}, \text{mad}, \text{duṣ}, \text{sādh}, \text{vardh}, \text{s'ubh}, \text{ruc})\ ṇi \\ \text{sah}, \text{tap}, \text{dam} \\ \text{jalp}, \text{ram}, \text{damp}, \text{yu}, \text{du}\ (n \to ṇ) \\ \text{sam}\ (\text{krand}, \text{kraṣ}, \text{karṣ})\ \text{vi}\ (\text{bhiṣ}) \\ \text{Jan}_3 + \text{ard}, \text{madhu}_3\ \text{sūd}, \text{vill}_2 + \text{vi} + \text{vās'}\ \left[\begin{smallmatrix}\text{kula}\\ \text{kāṣṭha}\end{smallmatrix}\right]_{1}^{3} + \text{dam} \end{array} \right\} \text{yu} \to \text{ana}$$

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23. Kv. Aṣ - Ṇo3
24. 3.1.141
25. Kv. Aṣ - Ṇo3
26. 3.1.142
27. 3.1.143
28. K
29. Kv

(ii)

Place

- grah, manḍa
- ut (sah, vas, bhōs), sam (mard)
- ṇī (Rakṣa, śāu, vas, vah, śā
- naṁ ($R_V^{a'}$, yāc, vi+ā+hṛ, sam+vi, ā+hṛ, vṛaj, vad, vas
- vi (śā, ṣa)
- abhi+bhū, apa+rādh, upa+radh, pari+bhū+nic, pari+bhū

] ⌒ Nin

Here, a' = agent is non-conscious being.

(iii)

- R(i, u, ṛ)$_c$, jñā, prī, kṛ, prī (Rā), grah/house, gṛhā/name → gb
- P_y (pā, ghrā, dhmā, dhaṭ, dṛś)
- limp, vind, (dhṛ, pṛ, vid, ut+ej, śāt, saḥ) ṇic
- ni (limp) P_2 + vind
- dā, dhā
- R. [jval ---- kas], bhū
- pr (tan, śyai, rā, vyadh, āsu, saṃsu, atīṇ
  - avasā, ava+hṛ, lih, śliṇ, śvas, du, ni
- grah / water animal

] ⌒ ka, ⌒ śa, ⌒ ṇa, ⌒ ṇu, b

x ↑ etc.

16. Affixes : Denoting - Artist —

The affix 'ṣvun' is added after a root when the agent so-expressed denotes an agent.[1] This affix should be confined the roots 'nṛt', 'khan' and 'ranj'[2]. The nasal of the root 'ranj' is elided, simultaneously. According to Mahābhāṣya, only first two roots are governed by this rule.[4] The word 'śilpin' used there means the agent's remarkable ability in performing the act.[5]

Root 'gai' - to sing, takes the affix - ṇyat as well as 'thakan' in SSC.[6] The indicatory letter - 'ṇ' of 'Thakan' stands for acute-accent, which falls on the first vowel.[7]

After the roots - 'ohāk' and 'ohaṛ', the affix ṇyat is employed, when the agents so expressed are 'rice' and 'time', respectively.[8] Here, also we find the remarkable ability. ohāk + ṇyat = hāyan, a kind of rice which leaves-off all water.
ohaṛ + ṇyat = hāyan, a year which goes through all conditions. (it is not so with days and months)[9]

The roots 'pru', 'sṛ' and 'lu' take 'vun' affix to express that the agent performs the action very well.[10]

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1. 3 1. 145
2. KV
3. K 534
4. MS 409
5. Pa 536
6. 3. 1. 146
7. N. 537
8. 3. 1. 148
9. As 409
10. 3. 1. 149. Samabhihāre yuchannemitam
Sādhukāritvam. lak gyate. (K 535)
N 539

Mathematical · Representation —

(ṇip)	nṛt , khan, (Rañj → Raj)	⌢	Svun	a	EXCELLENT PERFORM-ANCE
	bru , sṛ , lṛ	⌢	vun		
	ohāk/rice , ohān/year , gai	⌢	ṇyaṭ		
	gai	⌢	Thakan		

Affixes in Benediction—

Āśiṣi ca (3.1.150)

Affix 'vun' is employed after all the roots, when intended meaning is that of 'be the agent of this action'. Thus it expresses the benediction.

Word- Āśiṣ means a sort of prayer. here - it is related to the action..

Mathematically —

[x^a ⌢ vun] [may he be]

T2795

4

MATHEMATICAL DEVICE AND PĀṆINI

Mathematical Device And Pāṇini

It will be very interesting to examine whether Pāṇini made any use of mathematical technique in compiling his great treatise on Grammar. We must not be surprised to notice the frequent use of symbols, sets, subsets and matrix etc. His language is full of technical terms. Some of them are well defined[1] by him, while others seem to be traditional ones.[2] It becomes more important in the light of the fact that technical terms are unavoidable in a scientific study of any subject.

Well-known it is that Mathematics is full of symbols which are flexible in their nature for example, R stands for 'relation' as well as for 'ring' in modern Algebra,[3] for real numbers[4] in theory of real numbers and for radius of circum circle in trignometory This characteristic of flexibility is also found in the symbols of Aṣṭādhyāyī The indicatory 'ñ' in an augment (āgama) indicates the place of augment (i.e the augment will be added in the beginning) (1.1.46); in pratipadika it indicates that the affix ṅīp is added in feminine; in a set affix (i.e. luṭ, liṭ, ...) indicates that the 'l' of ārdhadhātuka affix is replaced by 'e' (3.4.79)

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1 Vṛddhi (1.1.1), guṇa (1.1.2), Saṃyoga (1.1.7), anunāsika (1.1.8), Saṃvṛta (1.1.9), Pragṛhya (1.1.11-19), ghu. (1.1.20) etc.

2 Abhyāsa (1.2.33) jati (1.2.58), Kāraka (1.4.23) Saṃjñā (2.1.3) etc

3 In Abstract Algebra — 16, 135, 21. Theory of Algebraic of Real Numbers)

4 Trigonometry and Mensuration

something 'big' is indicated by a symbol in Mathematics as well as in Aṣṭādhyāyī. In Aṣṭādhyāyī, this indication is done by indicatory letters, case affixes, accents, and pronunciation. The indicatory letters are known as 'anubandha' in Grammar. These are not only attached with affixes and roots but also with nominal stems. If we put the indicatory letters in brackets, then —

(a) affix — San = Sa(n)
Kyaṅ = (K) ya (ṅ)
Kyac = (K) ya (c)
ṇic = (ṇ) i (c)
tavya = tavya (No indicatory letter)

(b) roots — bhū = bhū
edha = edh(a) (a = grave a)
ata = at (ā) (ā - accented a)

(c) nominal stems — citraṅī[6] = citra (ṅī) (for ātmanepada - affix)
ekāgārikaṭ[7] = ekāgāri ka (ṭ) (for ṭhak)

Here, it is also clear that one affix or root may have more than one indicatory letter. These serve many purposes. Dr. Devasthali[8] had studied them in detail.

The case affix in ordinary used language of Sanskrit and the one used in Aṣṭādhyāyī do not express the same thing. They are indicatory in Aṣṭādhyāyī in some cases. Such case affixes are three — ablative, genitive and

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6 3 1 19
7 5 1 113
8 Anubandhas of Panini by Devasthali Dr. G V Poona University publication, Poona

Locative The ablative case i.e 5th case is employed to indicate or we may say to denote that the operation is to be understood to affect the state of what immediately follows the word with 6th case (i.e P_6)[9]
for example, rule (1.2.9) —

iko jhal = (iK + 6th case) jhal (San is regarded as Kit by anuvṛtti).
= (iK + 5th case) jhalādi (San is regarded as Kit
= iK (immediately after) jhalādi - - - - - -
= immediately after a root ending in 'iK', jhalādi San - - - - - -

Similarly,

ād guṇaḥ = 'a' + 6th case +, (guṇaḥ)
Here, ād = 'a' + 6th case
= immediately after 'a' letter / a word ending in 'a' letter.

But, where P_6 is used with 'Prak,' there P_5 is terminal-point and the sense is different from the one stated above for example, 'Prak Kadārāt Samāsaḥ (2.1.3.)
The genetive or possessive case (Sixth) stands for 'in place of'.[10] See, the example - (Rule 1.1.3)

iKa guṇavṛddhī = iK + 6th case, guṇavṛddhī
= iK (in place of) guṇavṛddhī
= in place of iK, guṇavṛddhī
= guṇa or vṛddhi is to be ordained in the place of (last) iK

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9 A. 1.1.67
10 A 1.1.49
11 3.1.92

the locative or the 7th case with a prātipadika suggests that the operation directed by that Sūtra is understood to affect the state of what immediately proceeds that term. For example, Rule 6.1.72 —

iKo yaṇaci

here, aci = ac + 7th case
= ac + immediately before
= immediately before 'ac'.

iKo = IKaḥ
= iK + 6th case
= in place of iK

yṇa = yaṇ (ādeśa)

Thus, the rule means that the 'yaṇa' comes in place of 'iK', if it is just before an 'ac'.

But if P7 is used in a rule in which anu-vṛtti of dhātu (3.1.91) is understood then P is designated as upapada[11]. For example — Rule — 'Pre ḍrudruṣ-Sruvaḥ (3.2.27) means that 'pre being the upapada' affix ghañ is added after roots dru, sru and sru. Thus —

pre = pra + 7th case
= pra being the upapada

Pronunciation is also of indicatory nature. A vowel is designated as 'it' if it is pronounced with the help of nose[12]. An 'it' letter is elided by rule 'tasyalopaḥ' (1.3.9). Thus, the nasal pronunciation of a vowel is for the sake of elision which itself indicates something.

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11 3.1.92

12 upadeśe (a) janunāsika it

for example, in the root edha, 'a' is grave accented and is nasal pronounciated. So it is elided, and remaining part edh' is 'anudāttet'. Thus elision of a grave accented vow-el is for the sake of ātmanepada affixes.

Pāṇini uses accents also to denote some-thing. circumflex accent is employed in a word which is required to be understood in following rules.[13] Ātmo-nepada affixes are employed after roots containing in-dicatory circumflex accent if the fruit of action accr-ues to the agent.[14] A root having an indicatory gra-ve accent takes ātmanepada affix.[15] Augment 'ām' is added optionally after letter 'ṛ' of the root, if it is exhibited grave accented in dhātupāṭha provided certa-in other conditions are satisfied.[16].

Besides this, we find that the concept of set, Matrix, ordered set, intervals etc are not only found but had been used very purposefully and efficiently in Aṣṭādhyāyī.

Gaṇa pāṭha is very important part of pāṇini school of Grammar. There are 261 ganas in it.[17] The first and the last of them are Sarvādi and kṣubhnādi ganas respectively.[18] These g-anas are nothing but the sets expressed in tubular form we are able to decide if a given element does or does not belong to it

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13 1.3.11
14 1.3.12
15 1.3.12
16 6.1.59
17 SK-4 -(827-940)
18 1.1.27 and 8.4.39

Let us examine the first set. It is —

1. Sarva 2. visva 3. Ubha 4. ubhaya 5. ḍatara 6. ḍatama, 7. anya 8. anyatra 9. itara 10. itat 11. tva 12. nema 13. sama, 14. sima 15–21. purva parāvara dakṣiṇottarā. purvādharā pi uyaravasthāyāmsamjñāyām. 22. svamajñāti dhanākhyāyām 23. antaram bahiryogopa-Samvyānayoḥ 24. tyad 25. tad 26. yad 27. etad, 28. idam, 29. adas 30. eka 31. dvi 32. asmad, 33. yuṣmad, 34. bhavatu 35. kim.[19]

This set is reffered as Sarvādi Guṇa in Aṣṭādhyāyī.[20] Hense, we may say that —

Sarvādi = { sarva, visva, ubha ..... Kim }

The scholars of Grammar regard that these 261 gaṇas are the only examples of 'gaṇas'. But if we use the word — 'gaṇa' in the sense of set, then there exist so many se-ts in Aṣṭādhyāyī. They are in set building form.

Consider the very first rule of it, which is — Vṛdhirād-aic. (1.1.1.) This means that 'ā', and 'aic = (ai, au) are called Vṛddhi. This means — Vṛddhi = { ā, ai, au }

Is it not a set? Similarly, guṇa, ghu, nuṣṭhā, kṛt, kṛtya etc. are examples of set. Some of them are different to some extent from these two standard forms. But it is rather a merit.

Consider another rule. (1.2.26). It is — "Rulo vyā-dhātu-halādeḥ sam s'ca." Its meaning is that the

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19 S.K-4 . 827

20. 1.1.14 ; 1.1.02

affix 'San' is regarded as kit if it is added after a ro-ot beginning with consonant, ending in anyone of 'sal' letters. Here, we find some conditions to be satis-fied by a root. Is there only one such root? No. There are so many roots, such as — likh, dyut etc.

Thus,— sralo nyupadhāt haladeḥ = after roots (---

$$\text{अ} = \left[\begin{array}{l} \text{र}: \text{ र is a root} \\ \text{ending in consonant, begin} \\ \text{-ning with consonant and} \\ \text{having either 'i' or 'u' as} \\ \text{penalti made letter} \end{array}\right] = \left[\begin{array}{l} \text{likh, dru, kak, kuk, nik,} \\ \text{s'uak, tak etc,} \end{array}\right]$$

So, it is obvious that these roots consti-tute a set and it is expressed in set-building form.

Thus we can say that word 'Gaṇa' is understood in the sense of set expressed in tabular form. But Pāṇini has ~~extract the ex~~ used the set-building form and a third one also; where, though the mem-bers are mentioned in different rules, yet they fo-rm the set (like, Kṛtya, taddhita, Kṛtya ..)

Let us come to dhātupāṭha. What are the adādi class, juhotyādi class, kriyādi class etc? Are they not the subsets of dhātu defined by "bhūvādayo dhātavaḥ" (1.3.1.)? Yes, they are. Similarly, 'pūrvādinava'[21] is a nine membered sub-set of sarvādi set. In the same way, tyadādi also is the subset of sarvādi set. Thus, sub-set is used at least three times[22]

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[21] 7.1.16

[22] 1.1.72, 1.1.74; 7.2.102

Similarly, 'Gopuvanādi' sel is the subsel of 'vidādi' set[23]

Here, again we find a technical term for subsel and it is 'antar-gaṇa'.[24] This designation points out the clearness of concept and height of standard.

Some gaṇas are called - 'ākṛti-gaṇa' What does it stands for? It means that every member of such a set is not enumerated in the gaṇa. Where ever we find required condition fulfilled, we include that in gaṇa. When we are unable to count all the elements of subset or of a set, and the field of usage of language being limitless in time and place, how one can come to an end while counting the members Thus, it is infinite set, Set of roots and nominal stem (prātipadika are also the example of infinite set All the rest of the sets are finite. Once again, we are attracted by the technical name- 'ākṛtigaṇa'. (=Infinite set)

We also find the typical examples of Null-Set i.e. of a set which does not consist of any member. There are some affixes such as 'kvip' and 'kvin' where nothing remains to be added after roots. Each and every letter of the affix is elided. The root, having taken such a affix is regarded as if it contains that for example;

tad + dṛś + kvip

= tā + dṛ (ś → k) + ø → means change)

= tādṛk

---

23 2.2.67, 4.1.104, S.K 4 — 831 and 835

24 S.K.-4 832, K-

Then 'śūdrāḥ' is regarded as is containing affix 'luk' Tt
-us luk has its existence but not represented by any
sound. Thus it is a example of Null-set

Il has been already stated in the discus
-ption of Subset that pūrvādi and tyadādi are the
subsets of Sarvādi set. Now, suppose there is no order
in that set, then may pūrvādi and tyadādi set have
definite members? No. If we want to express that—

tyadādi = { tyad, tad, ------ kim }

and pūrvādi = { pūrva, para ---- adhara }

Then, we are to accept the order of elements in sarva-
-ādi set. Similarly the ten Subsets of roots are possi-
-ble only after the existence of order in dhātupāṭha.

Similarly, 'tiṅ' and 'sup' are also the examp
-les of ordered sets.

We also find the use of matrix in Aṣṭādhyā-
-yī. Rule 3.4.78 (tip, tas, jhi -----) means that 'laṭ' is
replaced by tip, tas, jhi ------ tiṅ. These affixes are ca-
-lled tiṅ. Rule, 'tiṅas trīṇi trīṇi prathama - madhya-
-mottamāḥ'[95] means that the every triad of tiṅ is called,
in order, prathama, madhyama, and, uttama.

Thus,

Prathama	—	tip	tas	jhi
Madhyama	—	sip	thas	tha
Uttama	—	mip	vas	mas

---

25 1.4.101

			ātām	
Prathama	—	ta	[illegible]	jha
Madhyama	—	thās	āthām	dhvam
uttama	—	iḍ	vahi	mahiṅ

And rule 1.4.102 means that members of triads are, in order — ekavacana, dvivacana, and bahuvacana. Thus —

	Ekavacam	Dvivacana	Bahuvacana
Prathama	Tip	Tas	jhi
Madhyama	Sip	thas	tha
Uttama	mip	vas	mas
Prathama	tā	ātām	jha
Madhyama	thās	āthām	dhvam
Uttama	iḍ	vahi	mahiṅ

Thus, we find that rule 1.4.101 divides the ordered set 'tiṅ' (of 6×3 = 18 members) in 6 rows and rule — 1.4.102 in 3 columns. Thus it is a 6×3 matrix. If we divide them again, according to pada, we will have two 3×3 matrics. Similarly, sup affixes make a 7×3 matrix. The addition of these affixes is like the addition of two matrices.

Pāṇini has a clear-cut concept of open and closed intervals. He uses words maryādā and abhividhi in these senses. Both of them are used together in the rule — ā maryādā abhividhyoḥ.[26] It means that the word ā, when signifying limit exclusive and limit inclusive, may optionally be compounded with a word ending in the

26 2.1.13 Vt

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fifth case affix, and the compound so formed is called avyayibhāva. Kāśikā gives examples as —

āpāṭali putraṃ vṛṣṭo devaḥ (maryāda, limit exclusive)

ā Kumāraṃ yaśaḥ pāṇinaḥ (abhividhi, limit inclusive)

These words are used at other places also.[27]

We also find the very intelligent use of letters to denote the digits. Theodor Goldstücker[27] says — Kātyāyana, moreover indicates (by expression bhāvyaa), and Patañjali expressly states that in those cases in which number of Sutrās comprised under an adhikāra did not exceed the number of the letters of the alphabate, a letter representing a numerical value (without, of course, being the bearer of a Swarita), was added to indicate the extent of the adhikār

The same method is applied by Āryabhata and others. But the value of a vowel or a consonant is different in Aṣṭādhyāyī. Indicating it, says Theodor Goldstücker[28] —

"..... and from example given by Kaiyata, we must infer that the numerical value of the letter was determined by the position it has in the Śiva Sūtras, since i is to him an equivalent of the figure 2. And this representation of figures by letters of alphabate discovers an additional interest from the circumstance that it is quite different from the

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27. Pāṇini — 67-8, Chowkhamba Sanskrit Series office, Varanasi - 1968

28 ibid. 67 8.

method we meet with on a later period of Hindu progress in Mathematics and Astronomy."

In foot-note he clears it as —

'Compare the system of Āryabhaṭṭa, who uses no vowels and nasals = 0; ka, ta, pa, ja = 1 kha, tha, pha, ra = 2; ga, da, ba, la = 3 etc.'

Thus, we find that Pāṇini's technique is based on some basic concepts of Mathematical standard.

5.

ADVANTAGES OF MATHEMATICAL REPRESENTATION

Advantages of Mathematical Representation

The scholars may or may not be agreed with the fact that Pāṇini used very intelligently some basic concepts of Mathematics. Even if they agree, mathematical representation of his rules hardly jus-tify its utility at the first sight.

Is it advantageous to any extent? Yes. Firstly, because an international language like - mathe-matical one plays a vital role in the rapid development of a subject. If we want to discuss the general and special trends in Sanskrit language (also, the langua-ge in general), it will help us to a great extent, bec-ause the field of Sanskrit is spread all over the world; so many languages are the medium of studying it. It's not the unknown fact that the development of sci-ence was slow before the use of international langu-age of symbols and equations. The use of this langu-age has set the pace of progress. The medium lan-guage becomes a hindrance to foreigners. Now, it can be minimised by the use of this Mathematical lang-uage.

Secondly, a given concept can be understood in a better way if it is examined from different angles, and in more than one language. A given rule will become more clear if it is explained in Sanskrit and is translated in Hindi and English. Mathematical representation presents us one more language, which is full of symbols and signs

The third advantage lies under the units of representation. In the present work, the unit of representation is quite different from a rule. To complete the meaning of a given rule, generally we understand some other words of previous rules thereby 'anuvṛtti'. We take such set of rules as the unit. One principal function is the deciding factor in constituting such a unit. Thus, this representation provides us a wider view of linguistic facts.

The last, but not the least advantage is the ability of representing some allied functions. For example, there are some particular cases of addition of san affix along with general one. Aṣṭādhyāyī deals them at one place. But we know that reduplication of one sy.llable takes place whenever the san is added. But Aṣṭādhyāyī deals with them at a distant part. In Mathematical representation, we give its glimpses.

It will, of course, will take a good time and require some labour to understand the new mathematical language. But, actually, it might be (not) with fairely grown-up peoples who are capable of following the meaning of rules by their great experience on the one side and find it very difficult to mould them by their own according to the new method. While for a new comer it is very easy thing, provided it is explained by able persons.

Any way, it should not mean that this representation is the replacement of Aṣṭādhyāyī. No, it is only a helping hand to it.

In the last, let me say that one likes one's own creation. But the work which is recognized by able persons, is essentially a work. Now, my work is open to the great scholars, who are the best judges of reality.

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